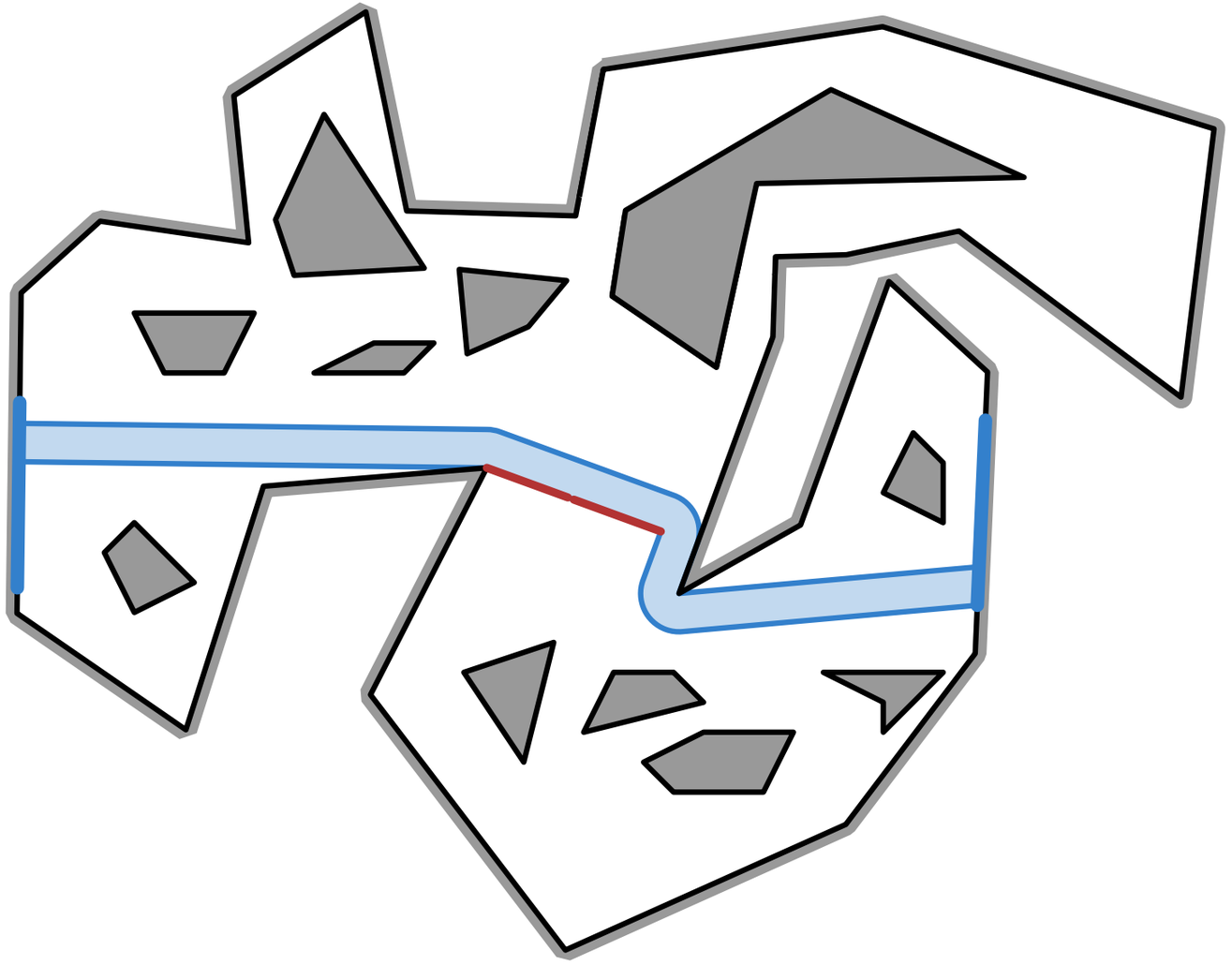
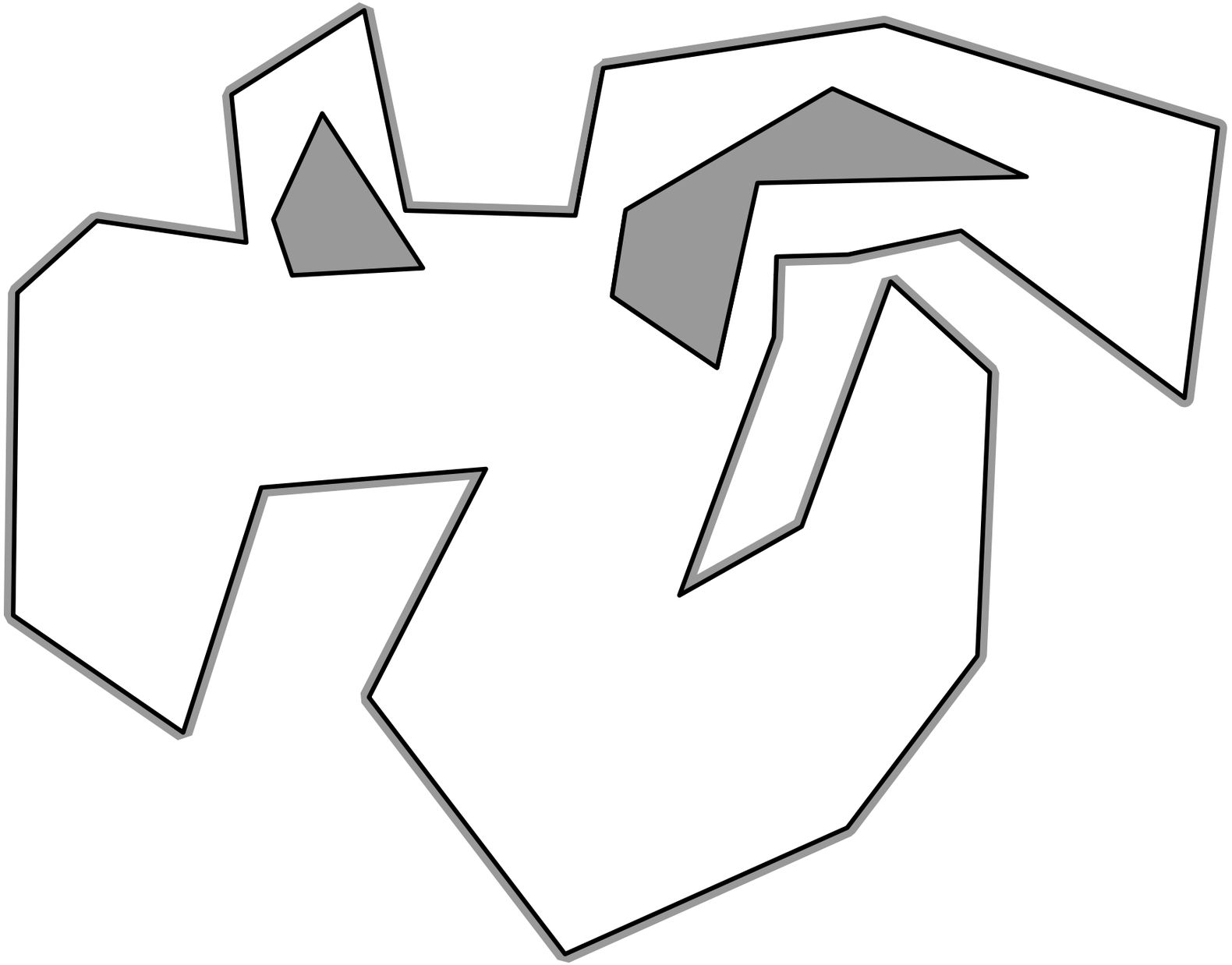
# Most vital segment barriers



Irina Kostitsyna Valentin Polishchuk Maarten Löffler Frank Staals

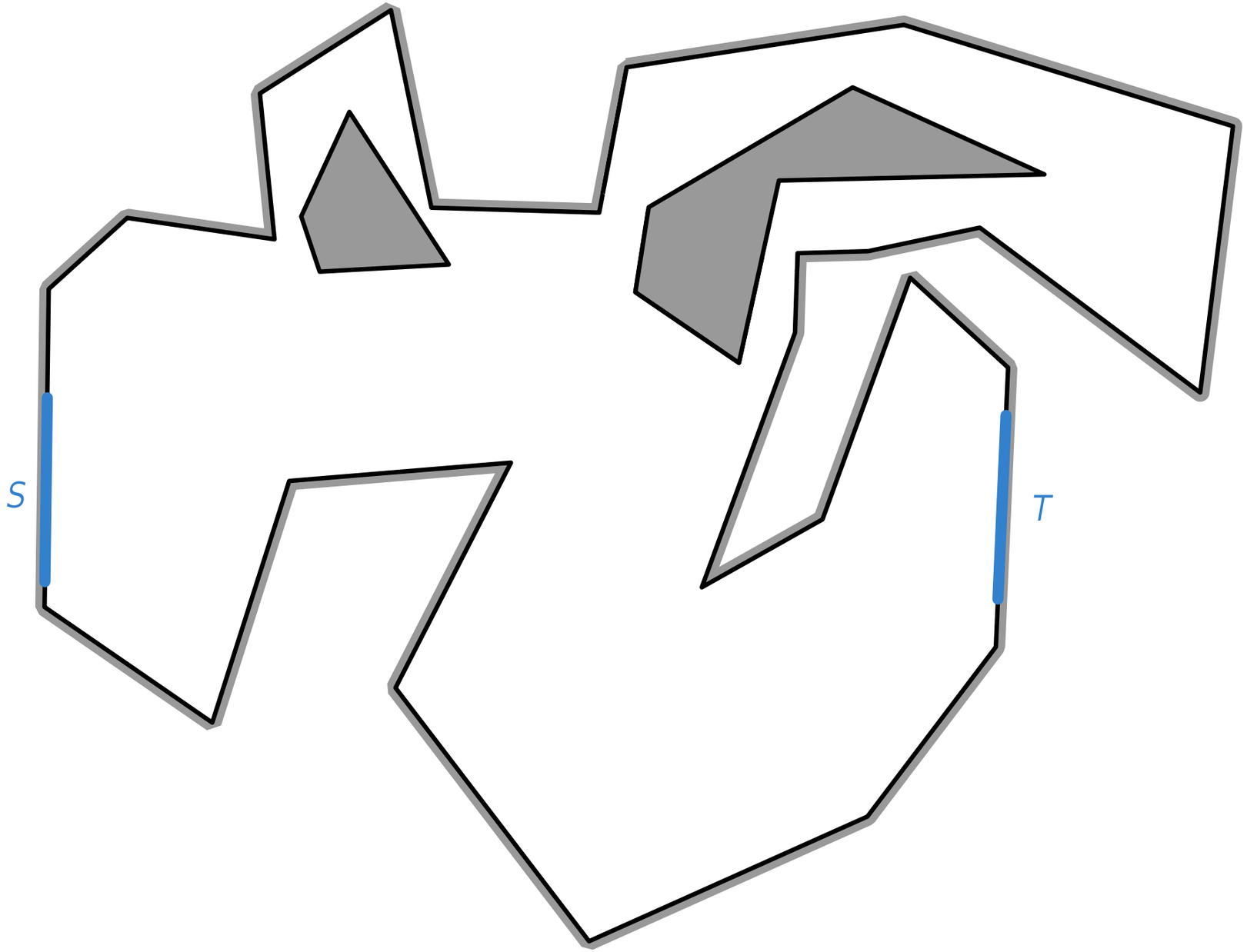
# The Problem



Given:

polygonal domain $P$

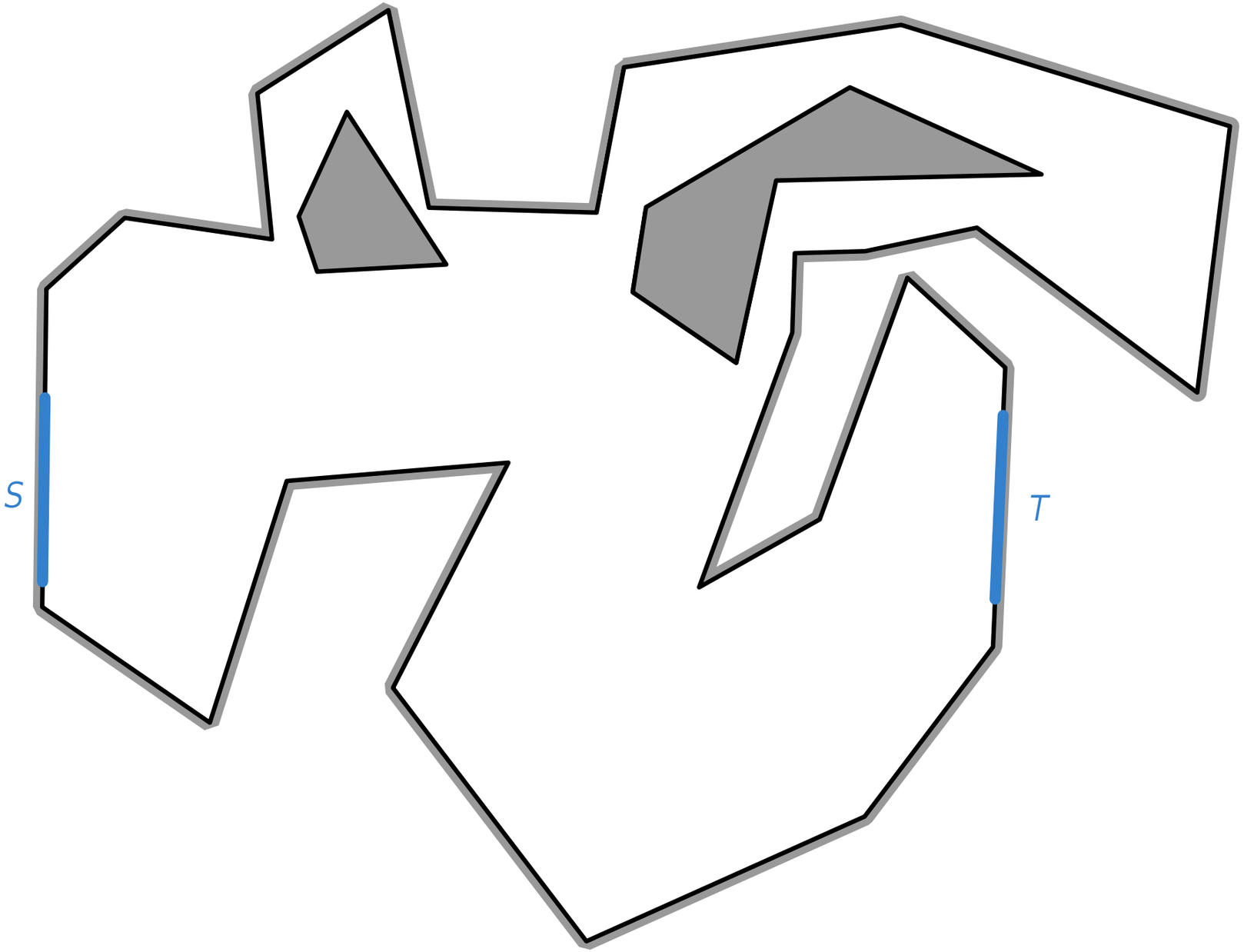
# The Problem




Given:

- polygonal domain *P*
- source *S*
- target *T*

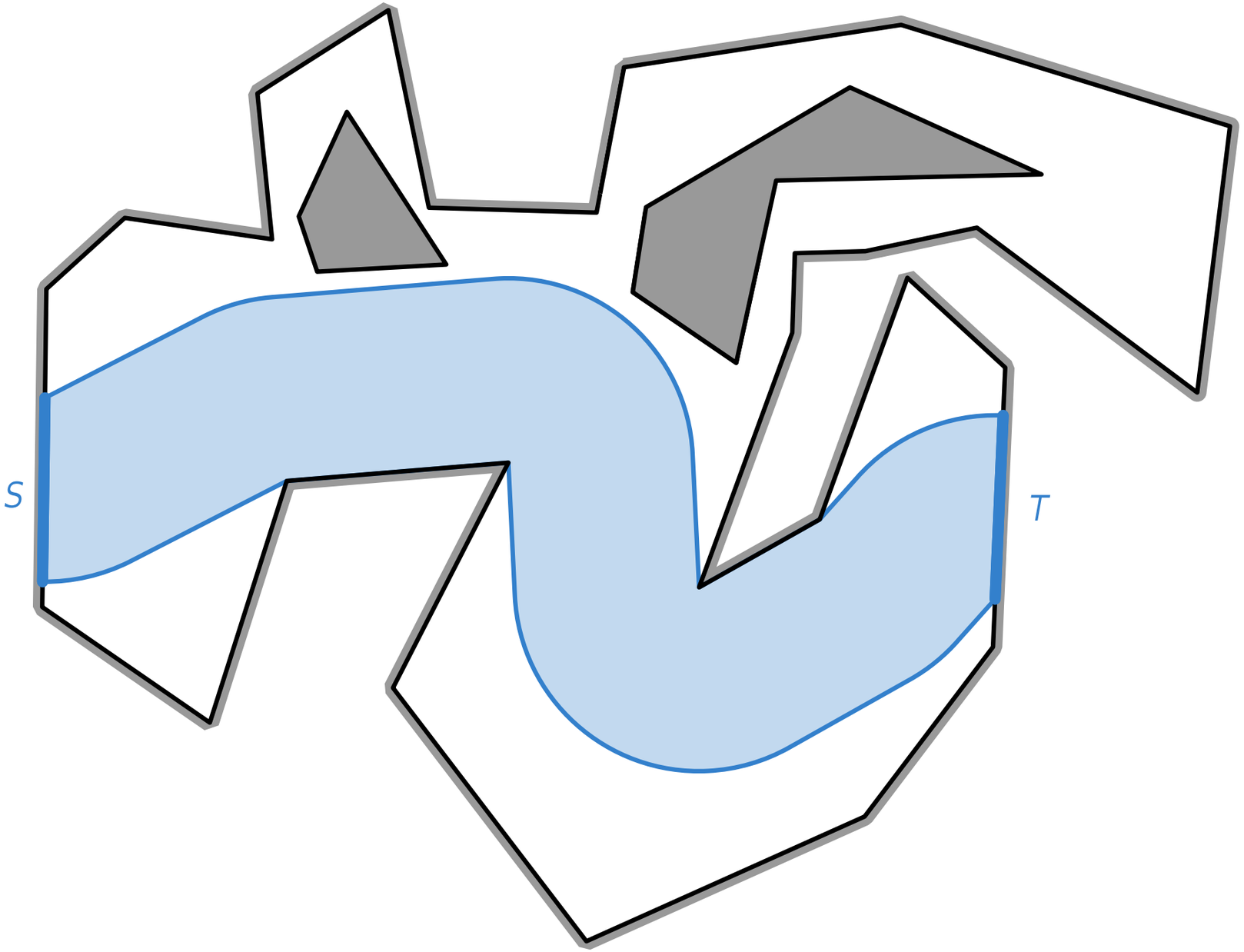
# The Problem




Given:

- polygonal domain *P*
- source *S*
- target *T*
- set of **barriers** *B*

# The Problem



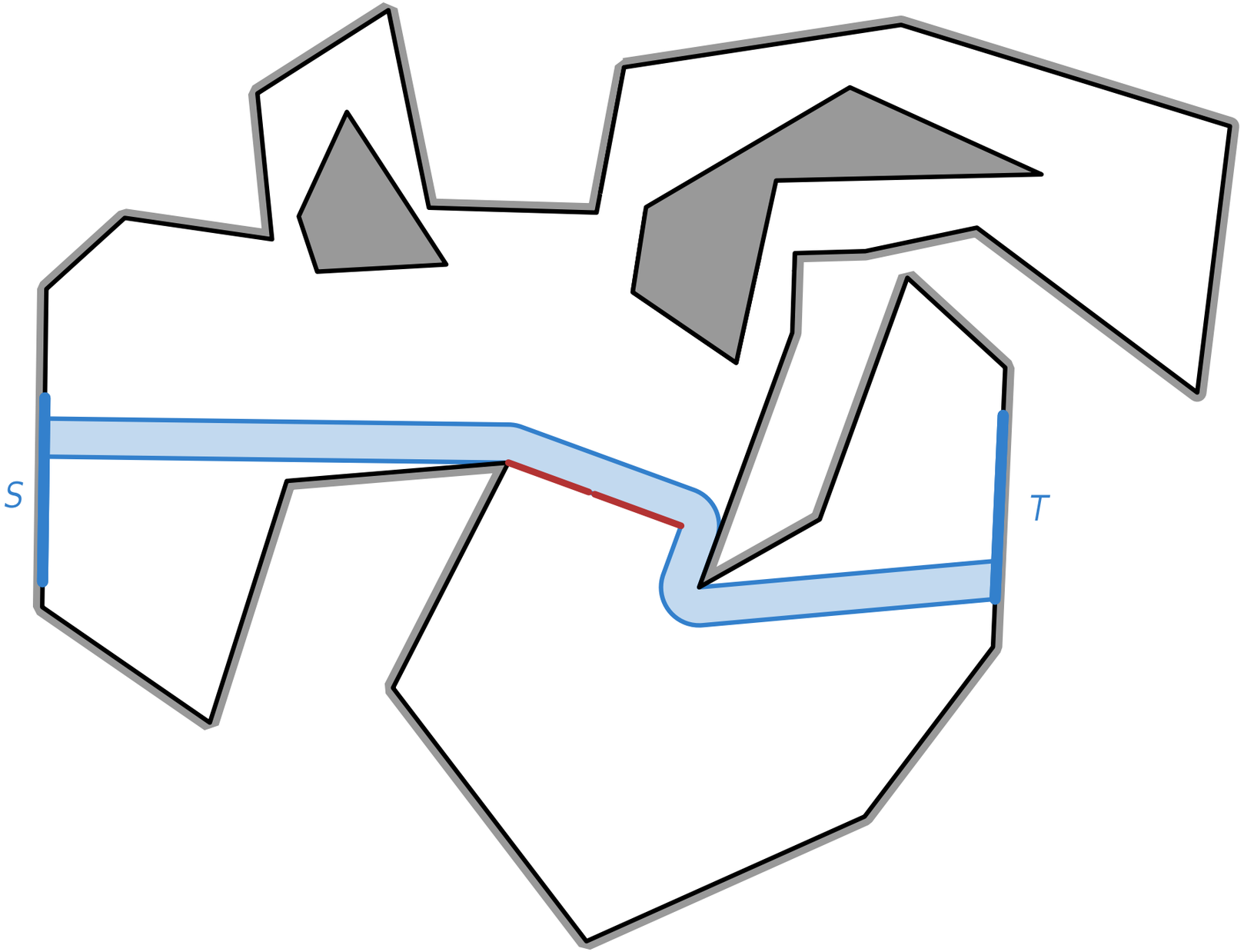

Given:

- polygonal domain *P*
- source *S*
- target *T*
- set of **barriers** *B*

Place the barriers s.t.

- flow from *S* to *T* is minimized

# The Problem



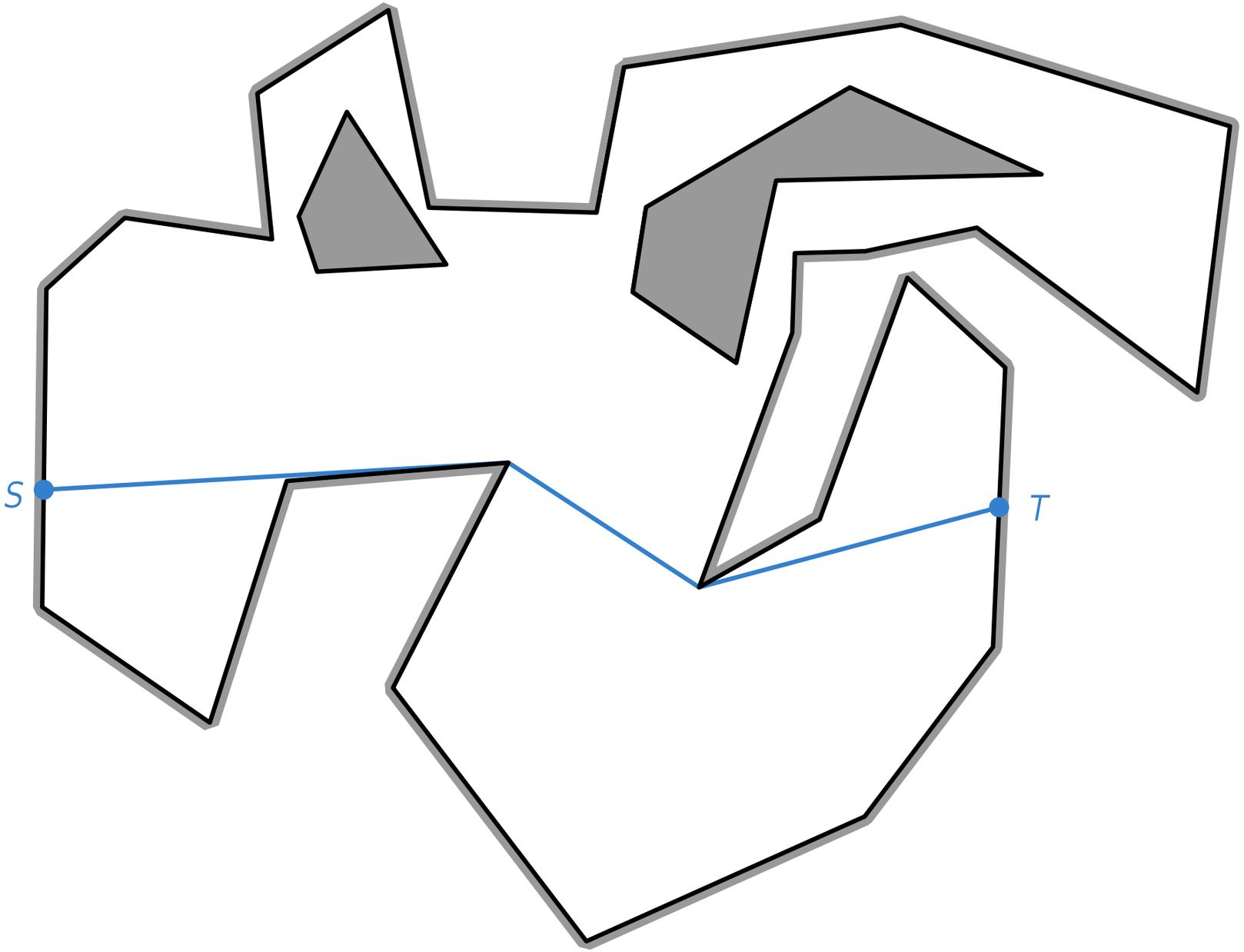

Given:

- polygonal domain *P*
- source *S*
- target *T*
- set of **barriers** *B*

Place the barriers s.t.

- flow from *S* to *T* is minimized

# The Problem




Given:

- polygonal domain *P*
- source *S*
- target *T*
- set of **barriers** *B*

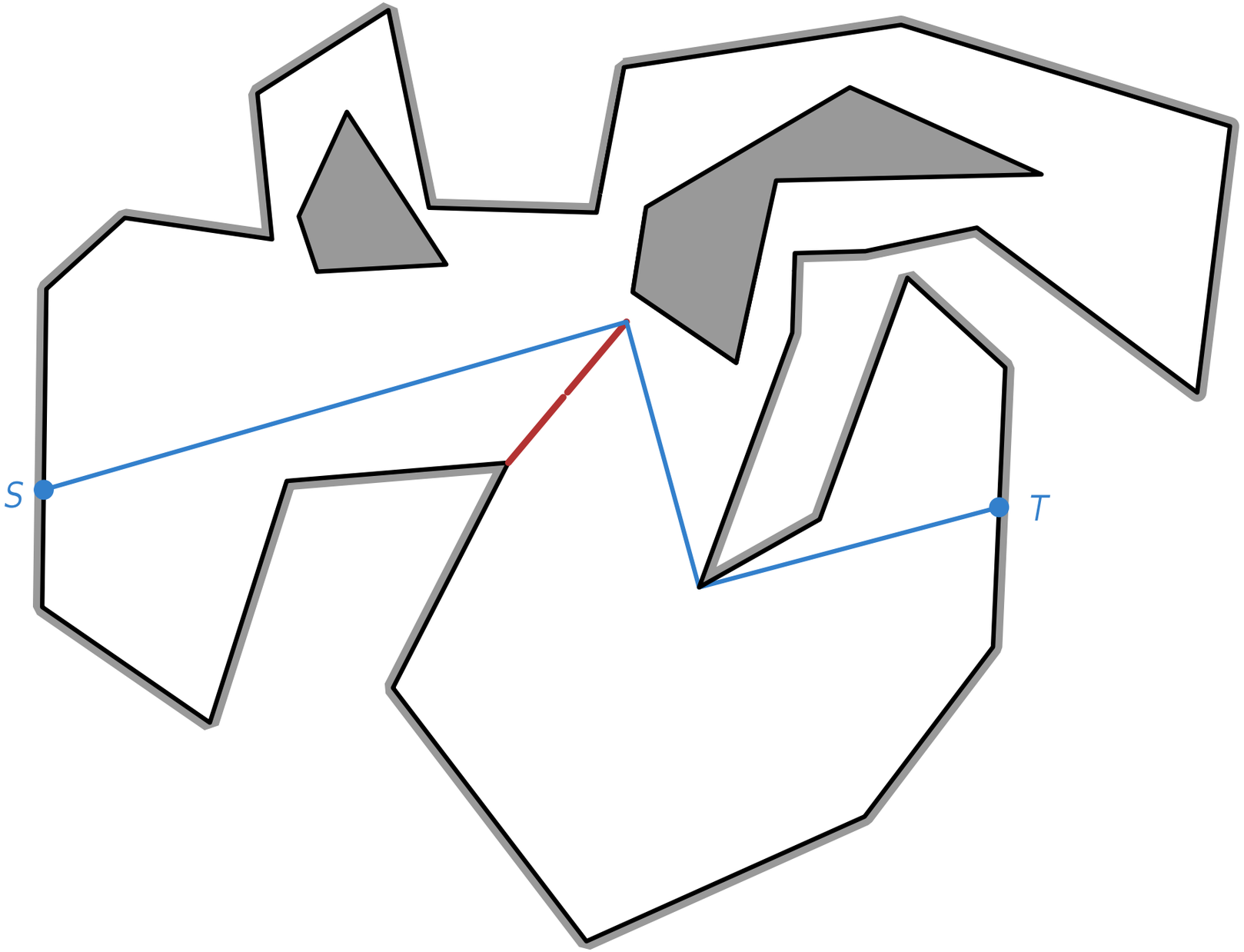
Place the barriers s.t.

- flow from *S* to *T* is minimized

or,

- length of the shortest path from *S* to *T* is maximized.

# The Problem




Given:

- polygonal domain *P*
- source *S*
- target *T*
- set of **barriers** *B*

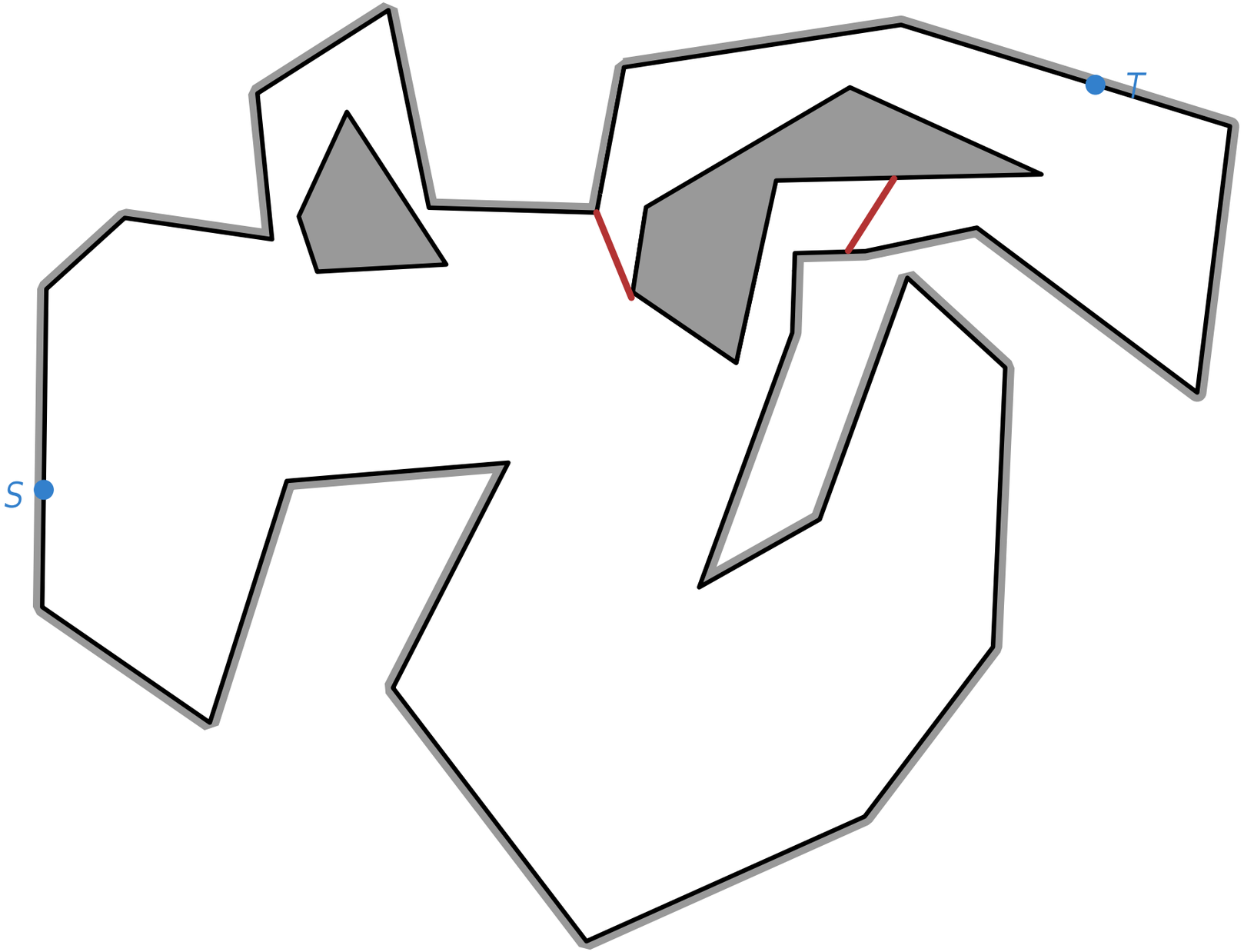
Place the barriers s.t.

- flow from *S* to *T* is minimized

or,

- length of the shortest path from *S* to *T* is maximized.

# The Problem




Given:

- polygonal domain *P*
- source *S*
- target *T*
- set of **barriers** *B*

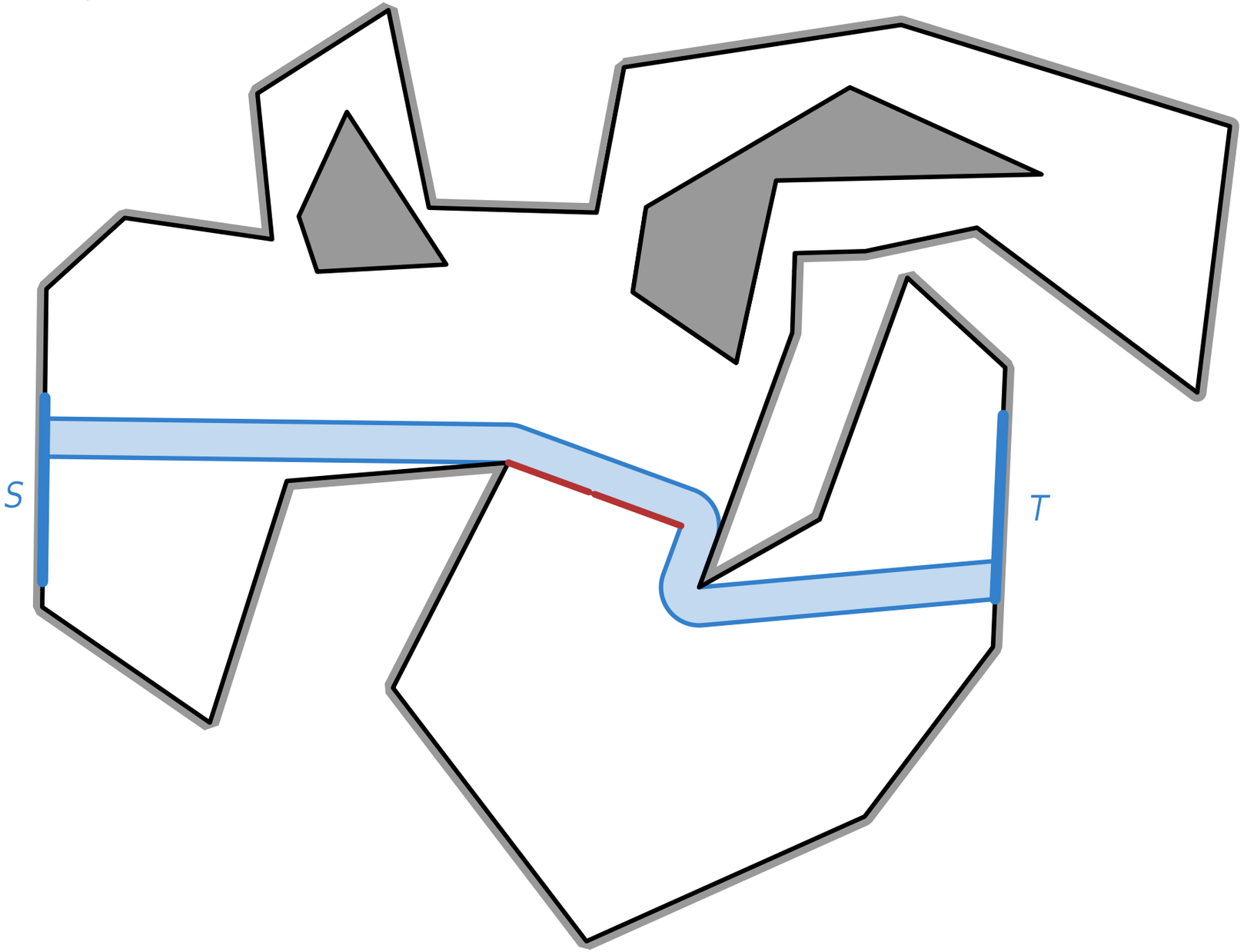
Place the barriers s.t.

- flow from *S* to *T* is minimized

or,

- length of the shortest path from *S* to *T* is maximized.

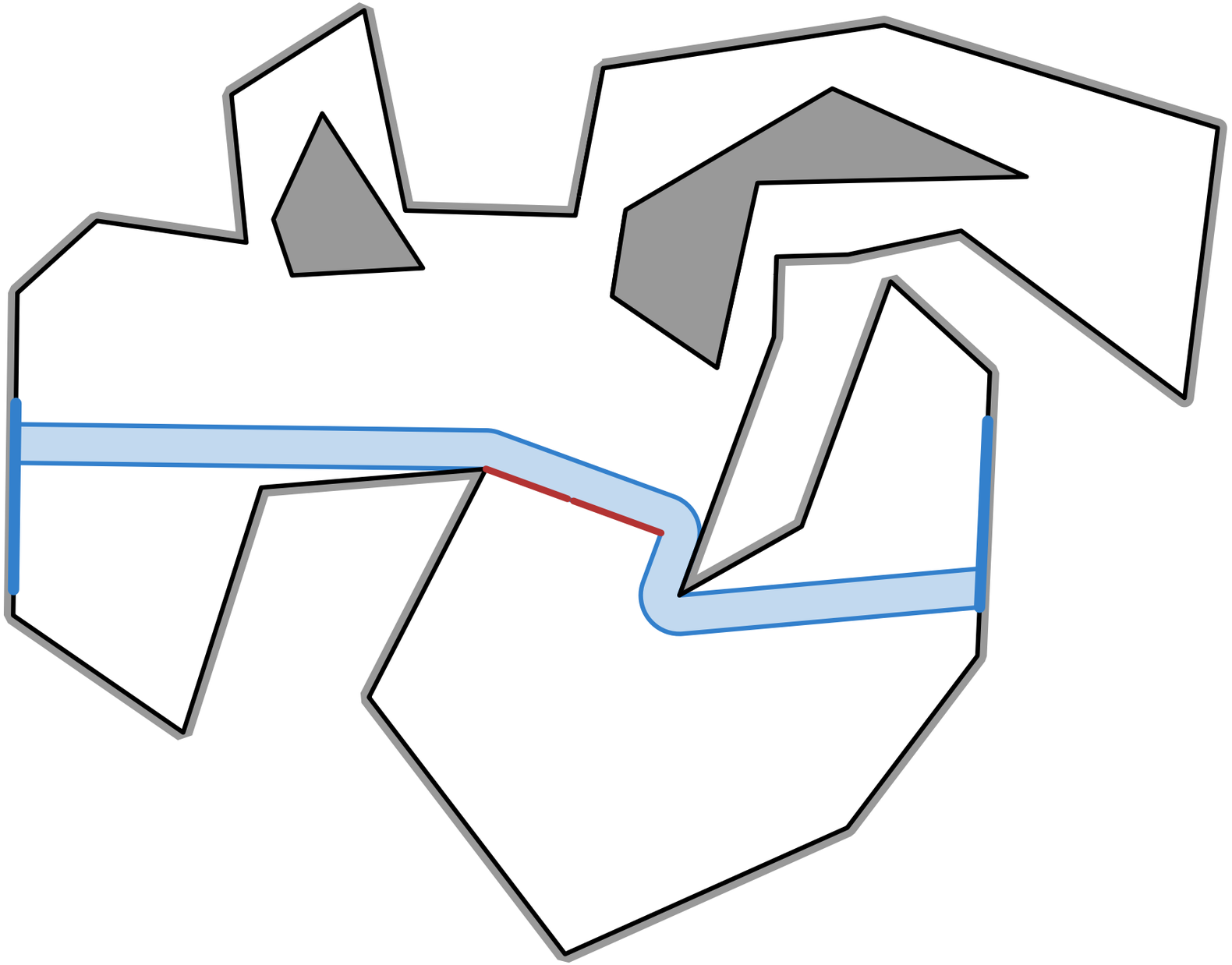
# Why?




Generalizes graph problems

Build dykes to delay floods etc.

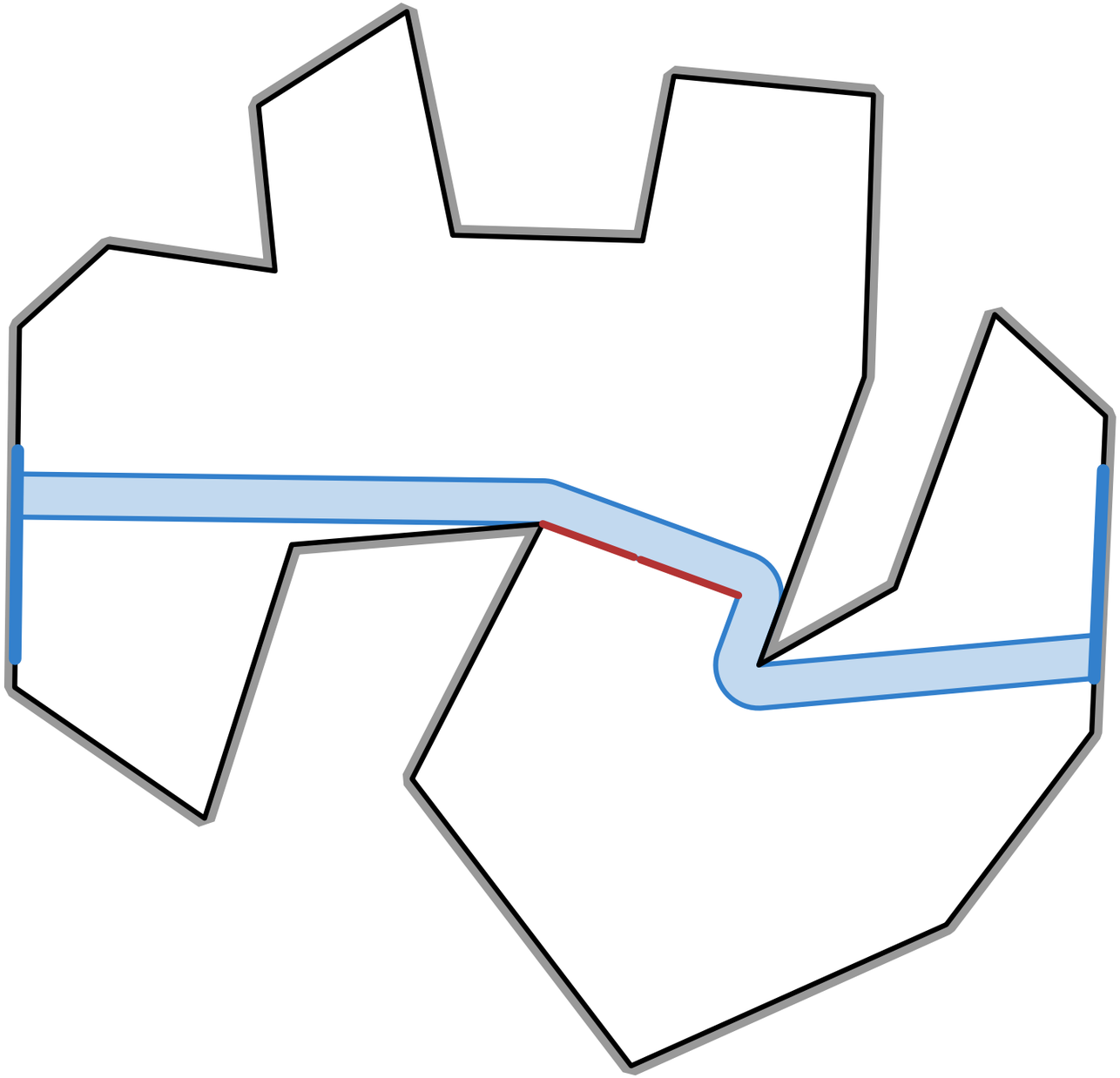
# Results



Depends on

- type of domain
- type of barriers
- #barriers

# Results



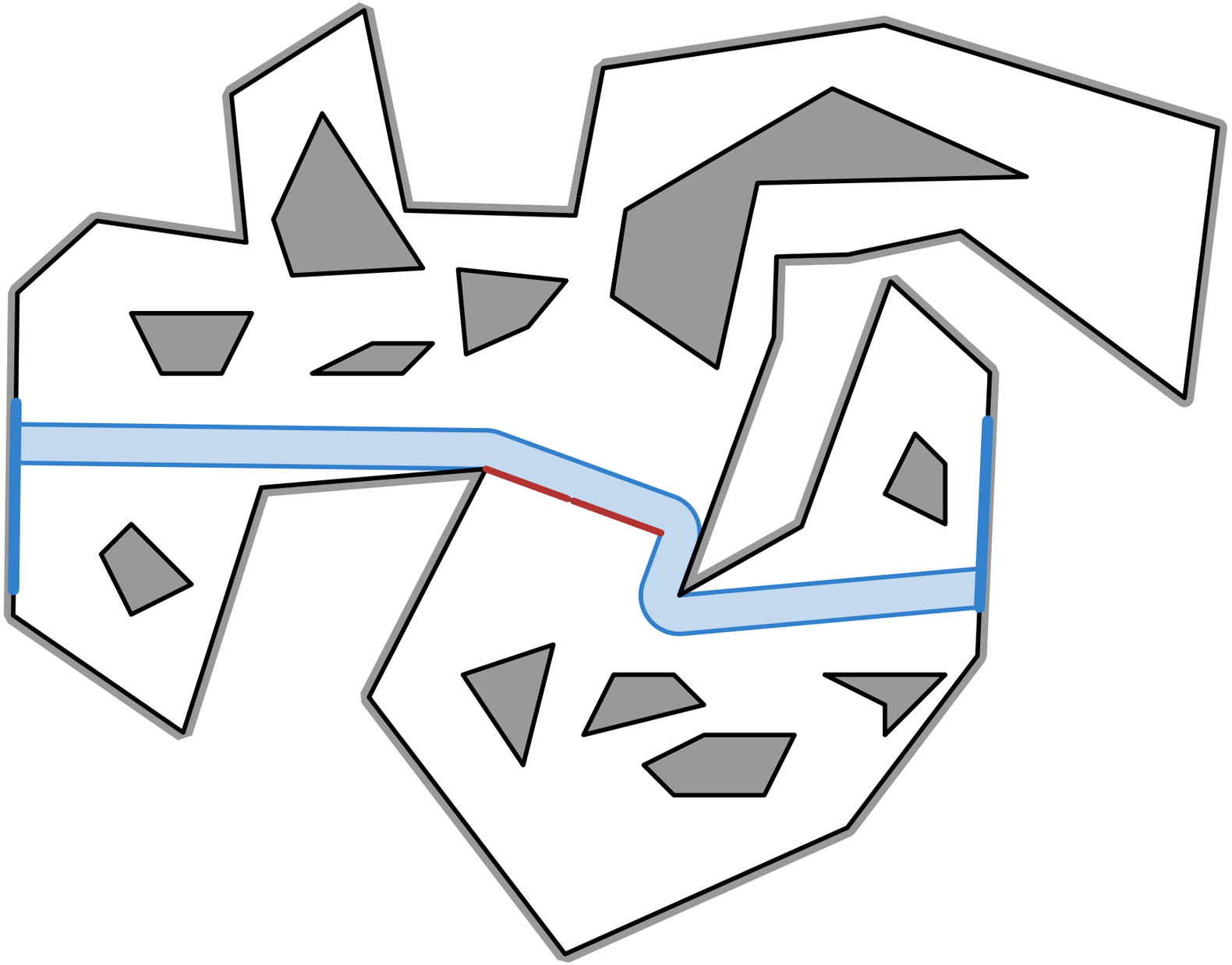
Depends on

- type of domain
- type of barriers
- #barriers

Flow

- $P$ Simple: $O(n)$ time

# Results



Depends on

- type of domain
- type of barriers
- #barriers

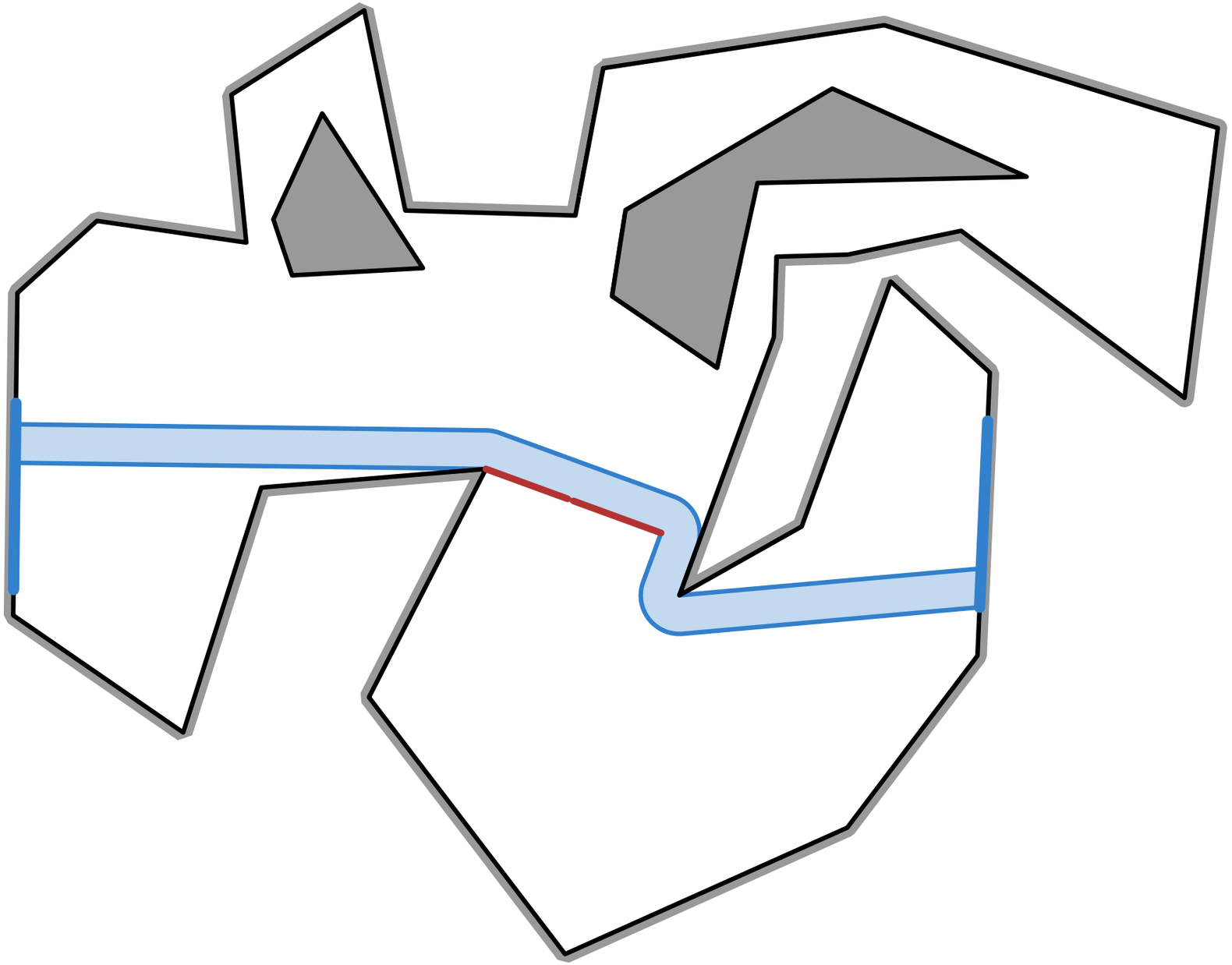
Flow

- $P$ Simple: $O(n)$ time
- $P$ polygonal domain

- many barriers
- diff. lenghts

} NP-hard

# Results



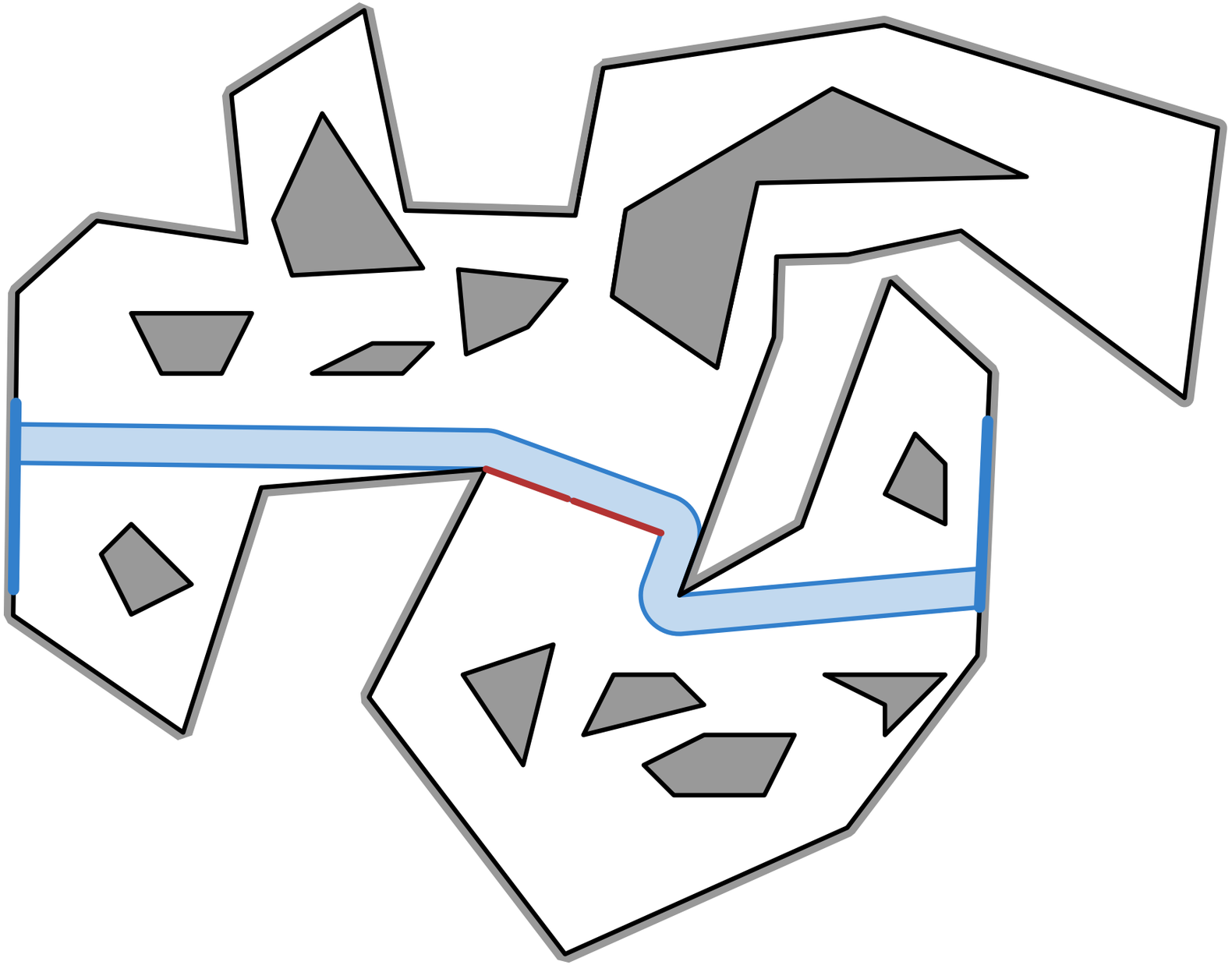
Depends on

- type of domain
- type of barriers
- #barriers

Flow

- $P$ Simple: $O(n)$ time
- $P$ polygonal domain
  - $O(1)$ holes
  - many barriers, diff. lenghts } weakly NP-hard

# Results



Depends on

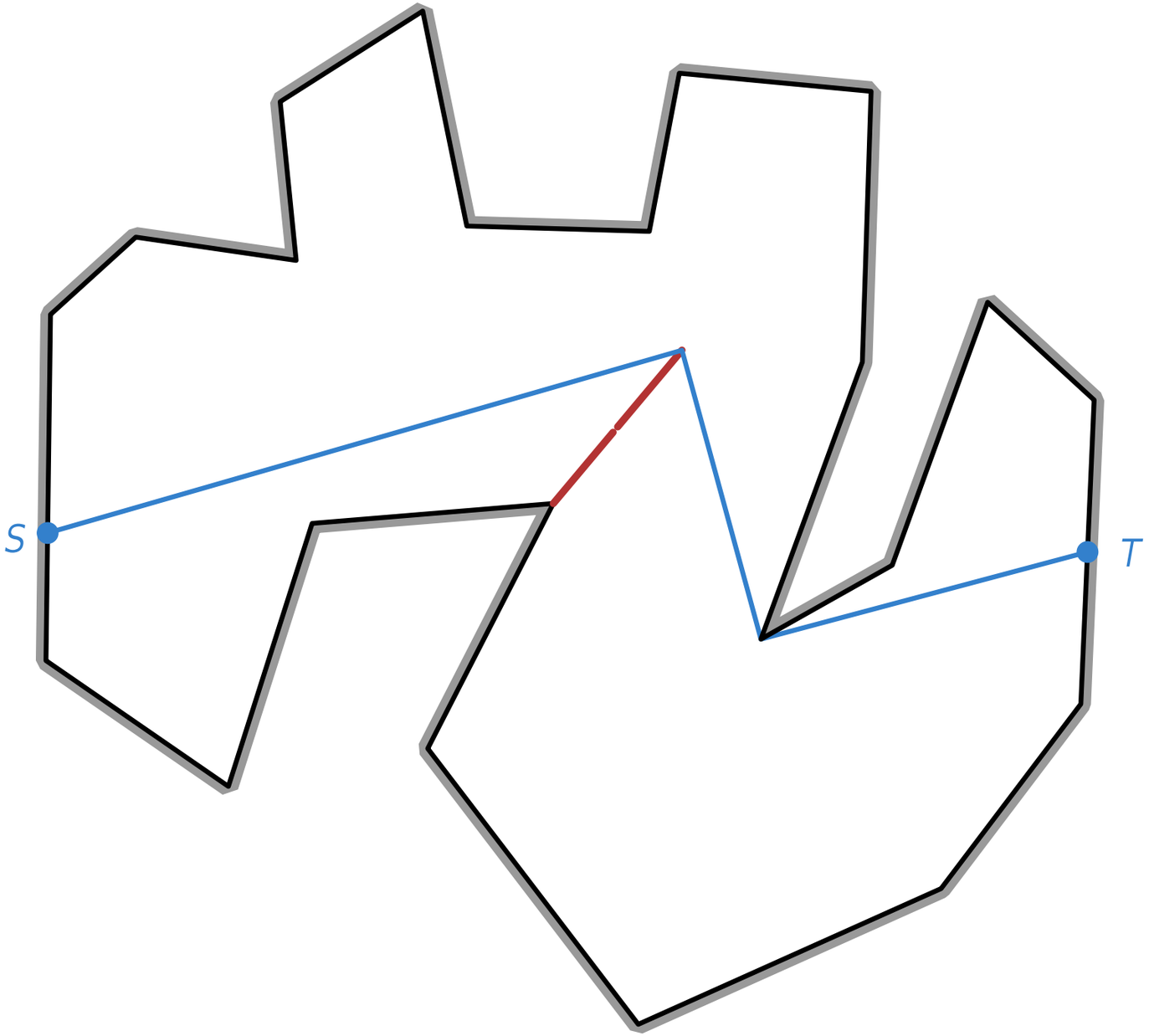
- type of domain
- type of barriers
- #barriers

Flow

- $P$ Simple: $O(n)$ time
- $P$ polygonal domain

many barriers, unit length } pseudo poly

# Results



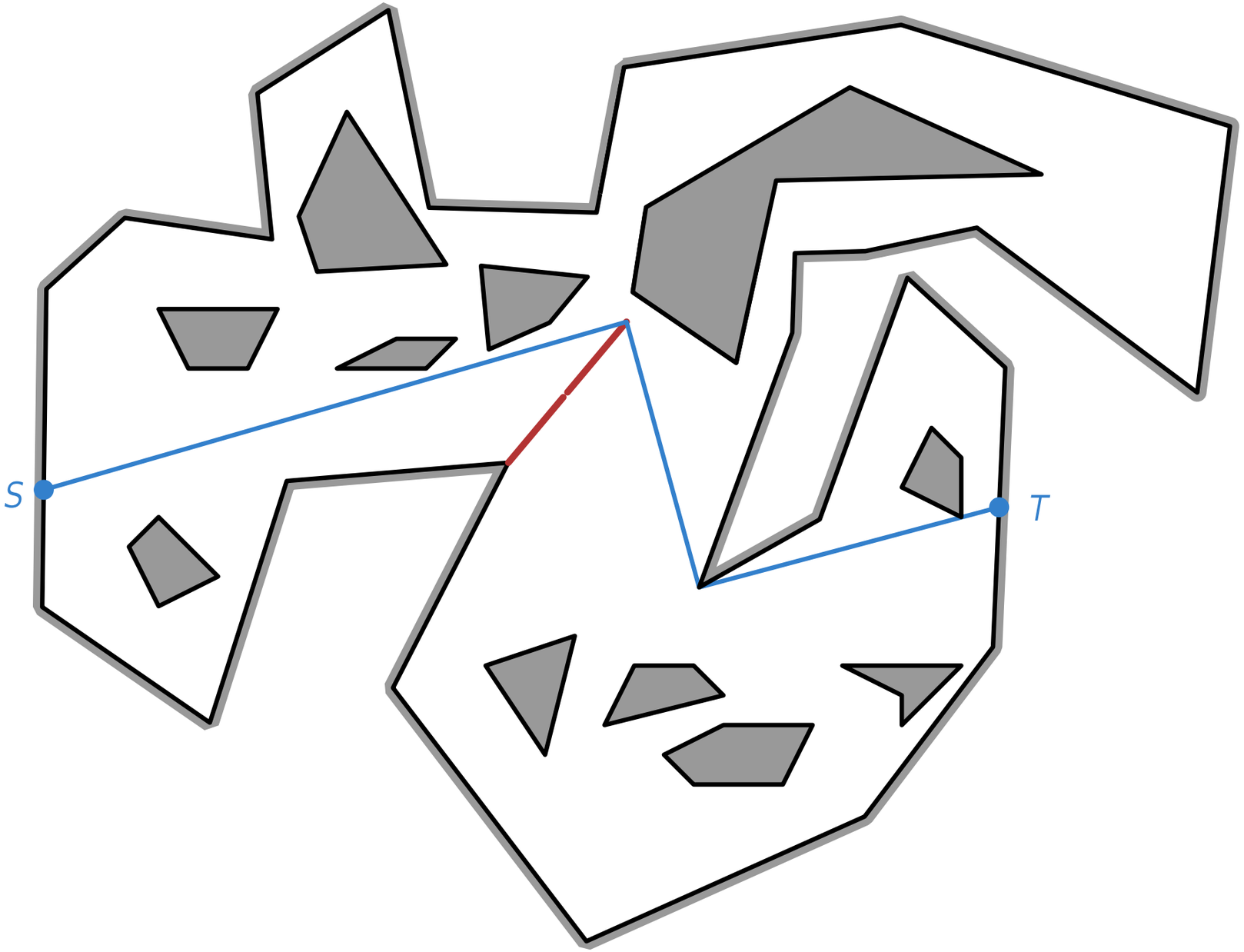

Depends on

- type of domain
- type of barriers
- #barriers

Path

- *P* Simple: $O(n)$ **time**

# Results




Depends on

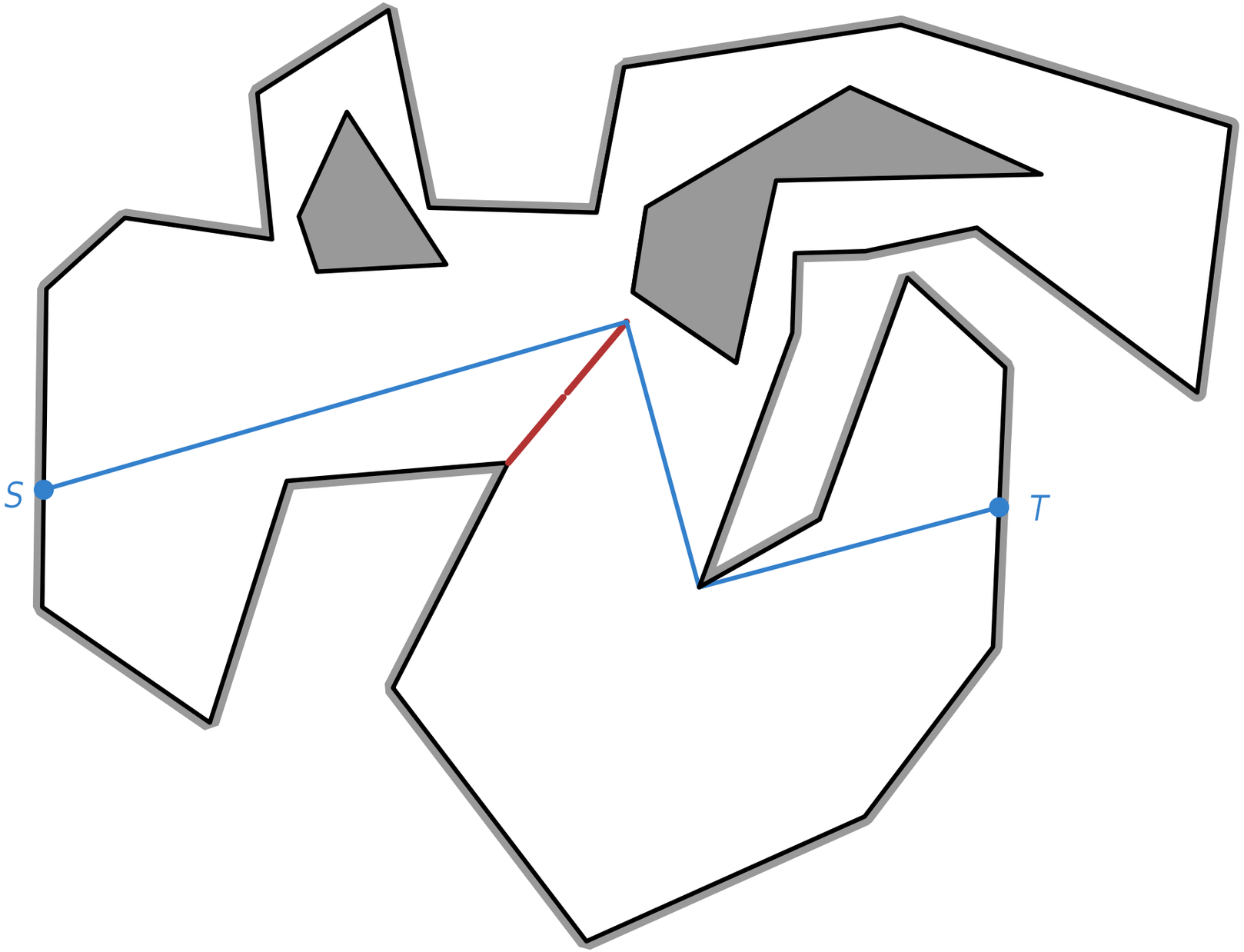
- type of domain
- type of barriers
- #barriers

Path

- *P* Simple: $O(n)$ **time**
- *P* polygonal domain

many barriers, diff. lenghts } NP-hard

# Results



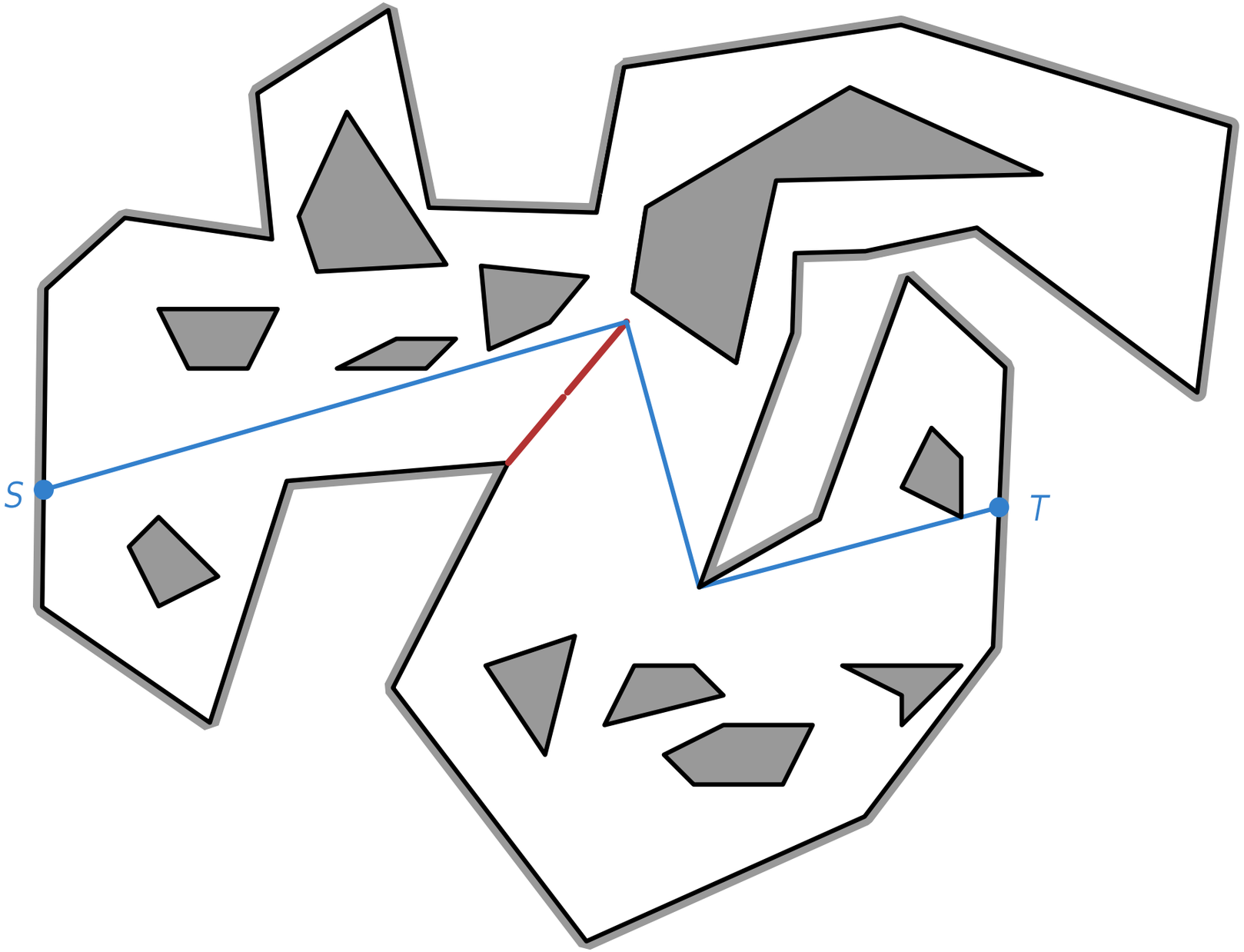

Depends on

- type of domain
- type of barriers
- #barriers

Path

- *P* Simple: $O(n)$ **time**
- *P* polygonal domain
  - $O(1)$ holes
  - many barriers, diff. lenghts } weakly NP-hard

# Results




Depends on

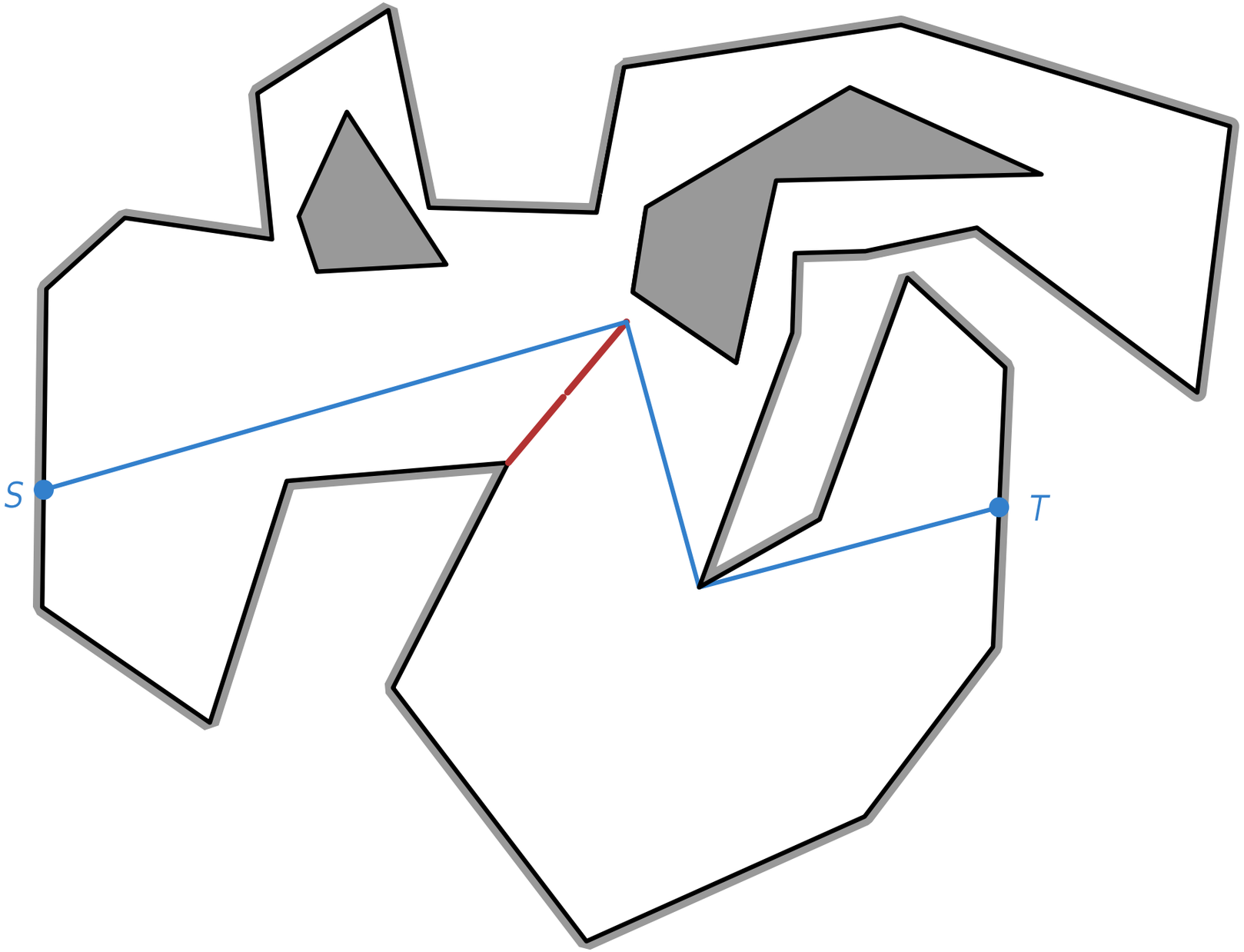
- type of domain
- type of barriers
- #barriers

Path

- *P* Simple: $O(n)$ **time**
- *P* polygonal domain

many barriers, unit length } weakly NP-hard

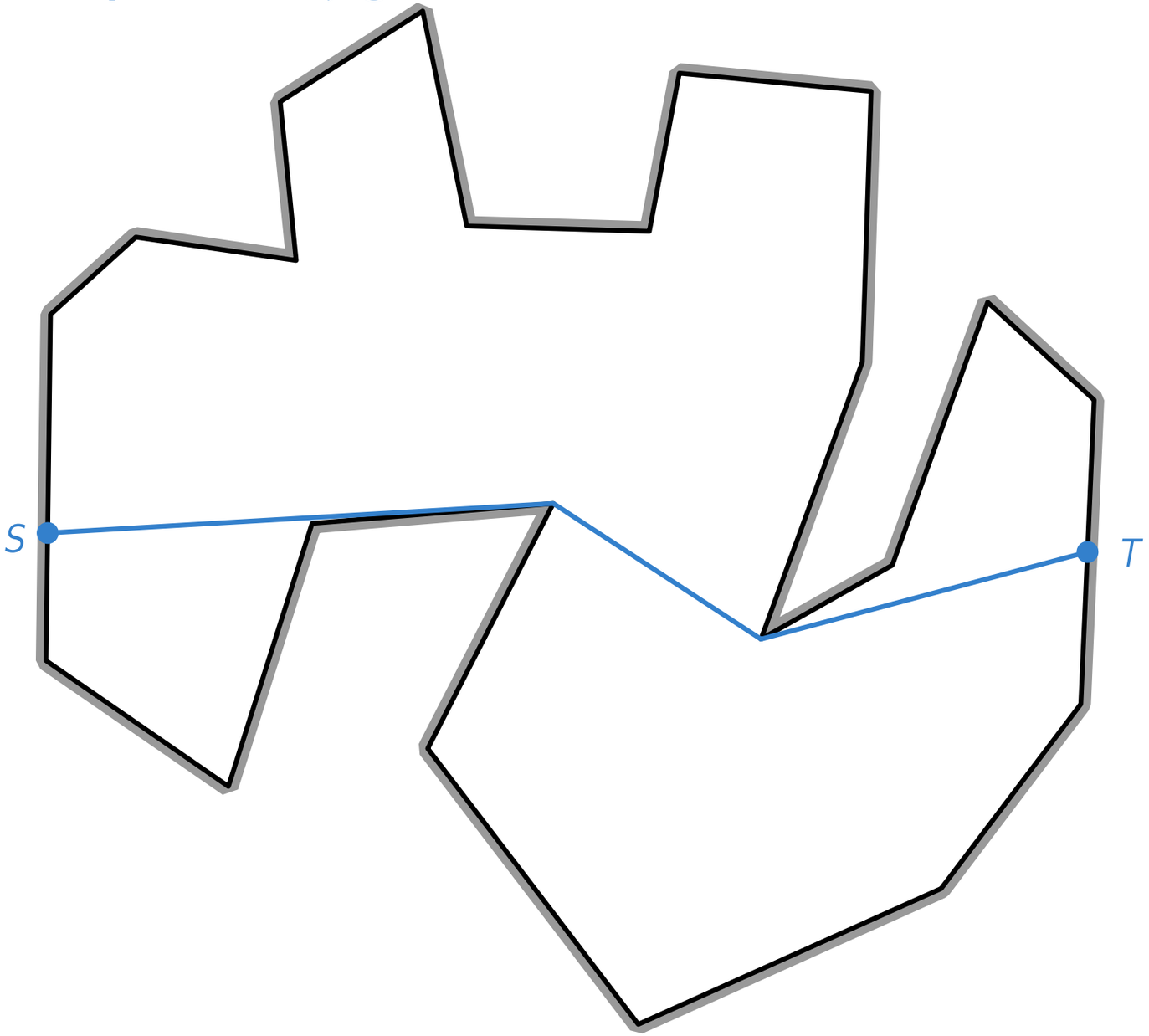
# Results




Depends on
- type of domain
- type of barriers
- #barriers

Path
- *P* Simple: $O(n)$ **time**
- *P* polygonal domain
  - $O(1)$ holes
  - many barriers } **OPEN**
  - unit length

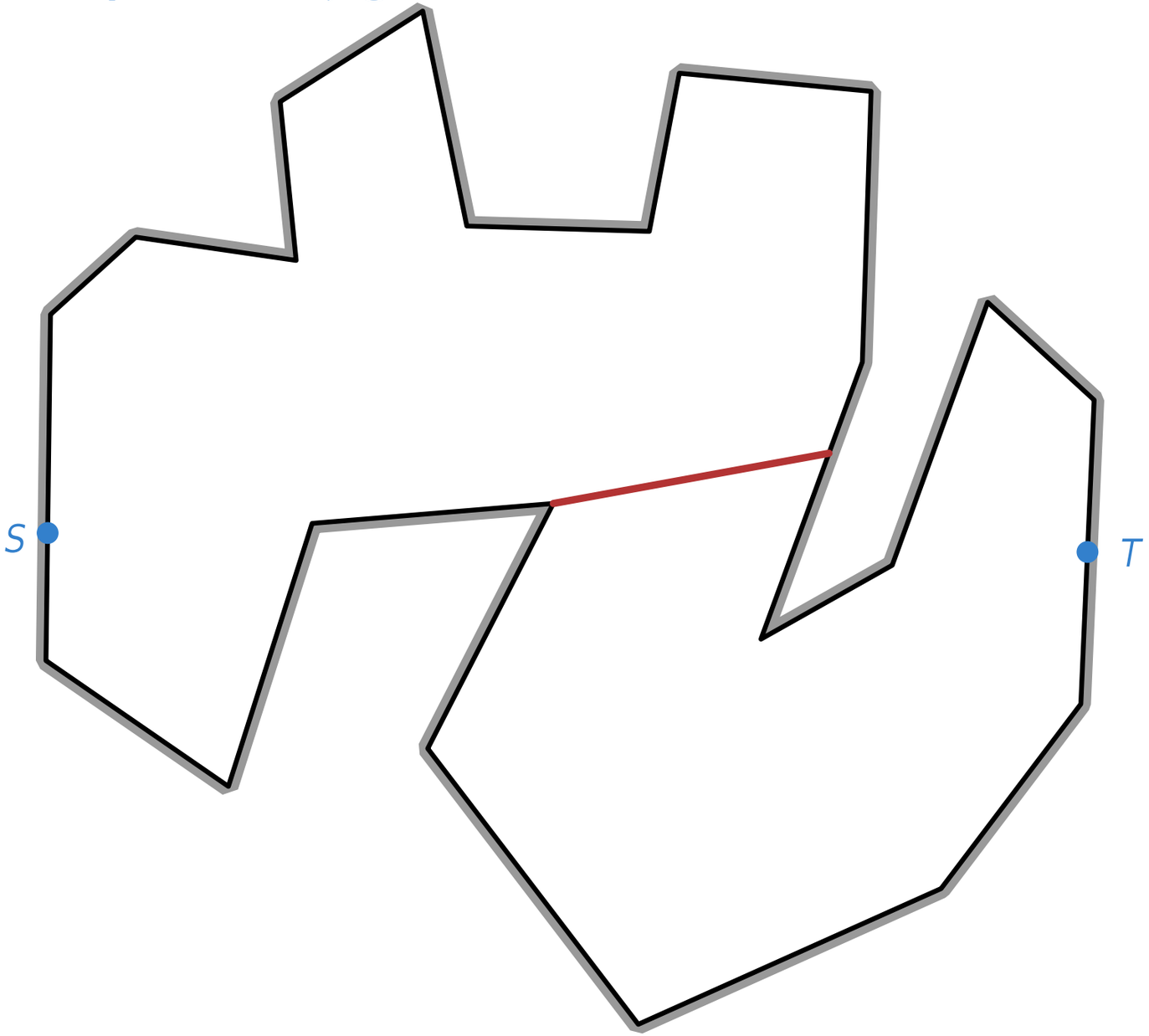
# Simple Polygon, 1 Barrier




Ingredients:

Test for complete blockage

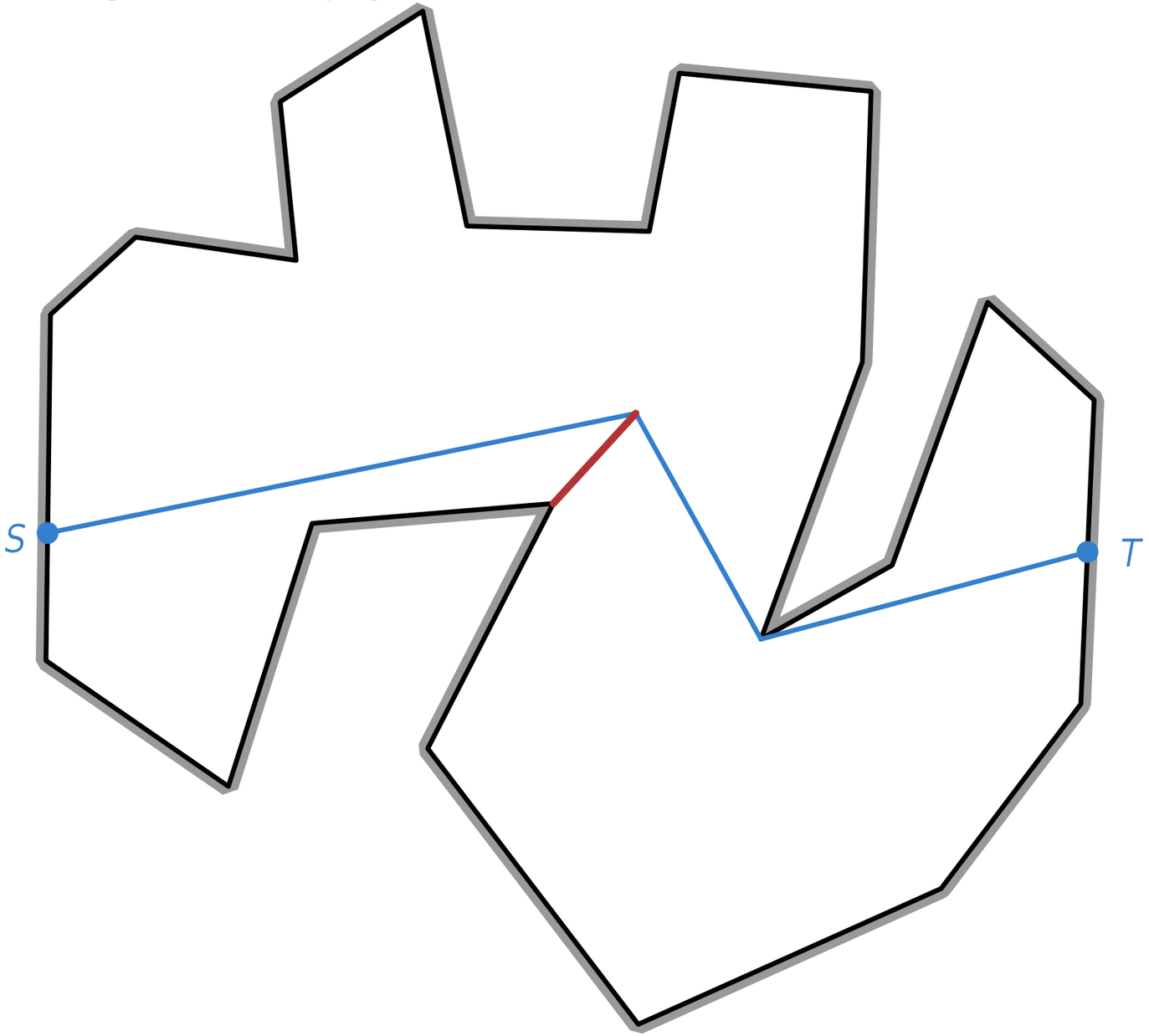
# Simple Polygon, 1 Barrier




Ingredients:

Test for complete blockage

# Simple Polygon, 1 Barrier

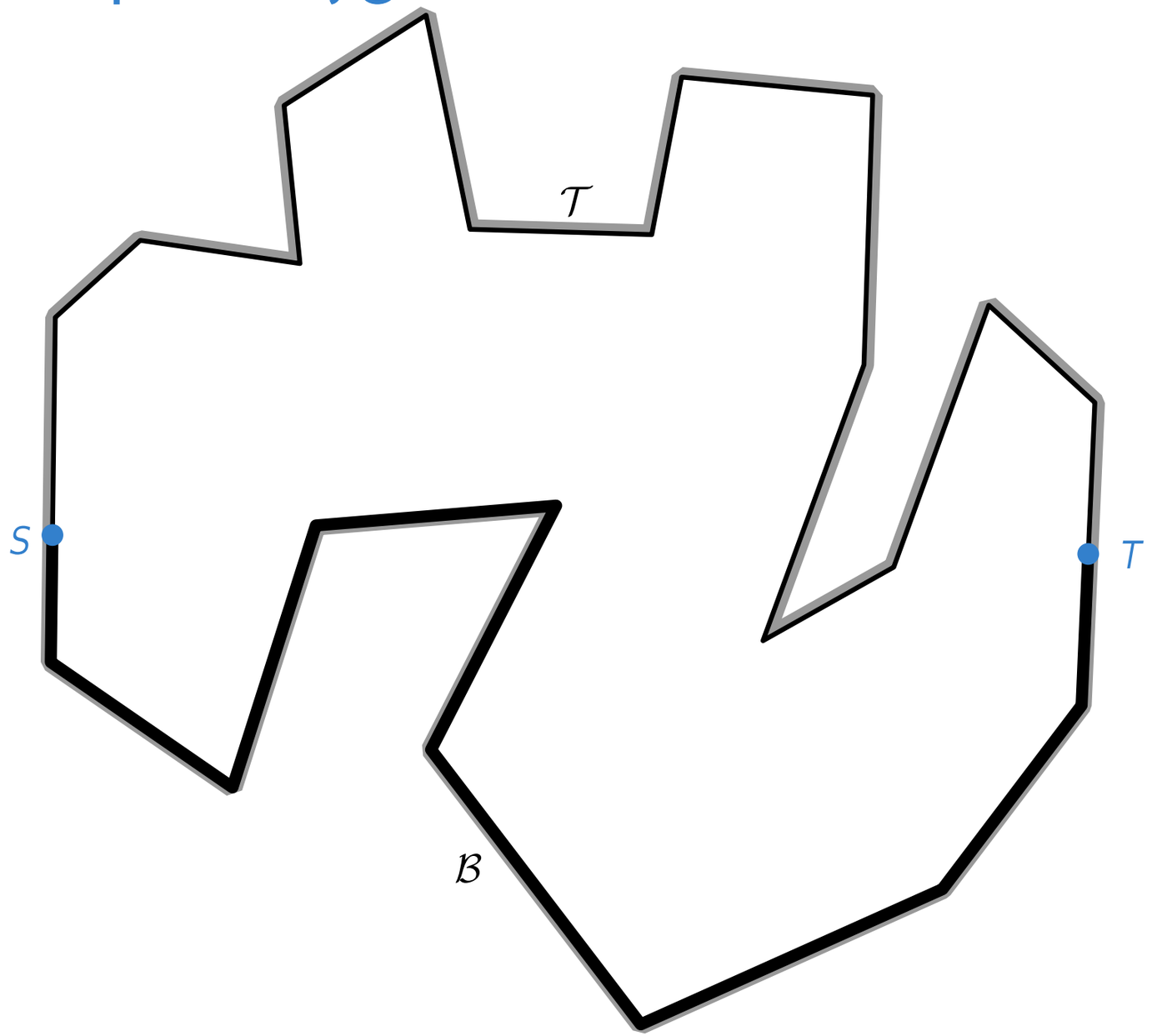



Ingredients:

Test for complete blockage

Maximize detour

# Simple Polygon, 1 Barrier

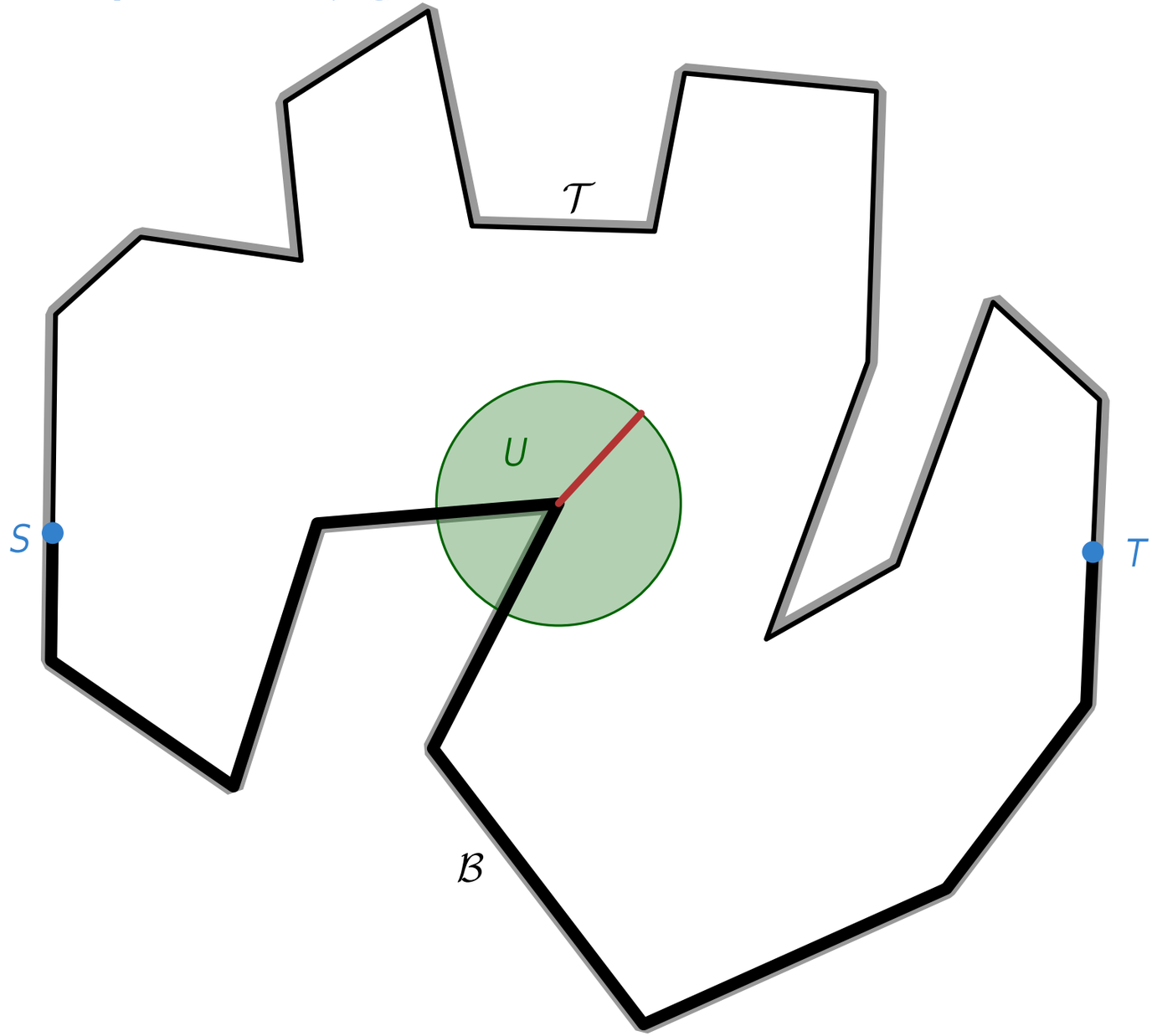



Ingredients:

Test for complete blockage

Maximize detour

# Simple Polygon, 1 Barrier

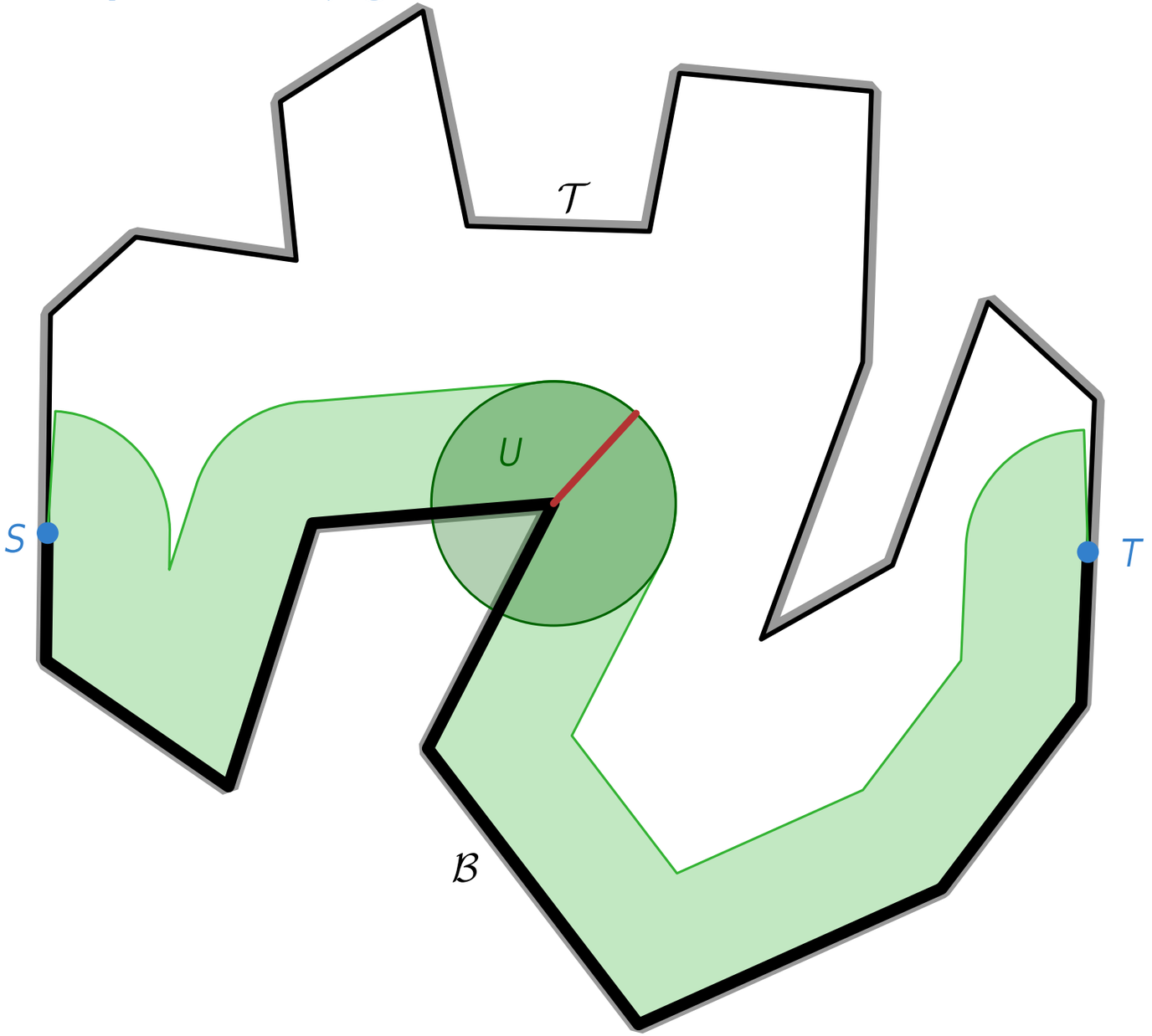



Ingredients:

Test for complete blockage

Maximize detour

# Simple Polygon, 1 Barrier




Ingredients:

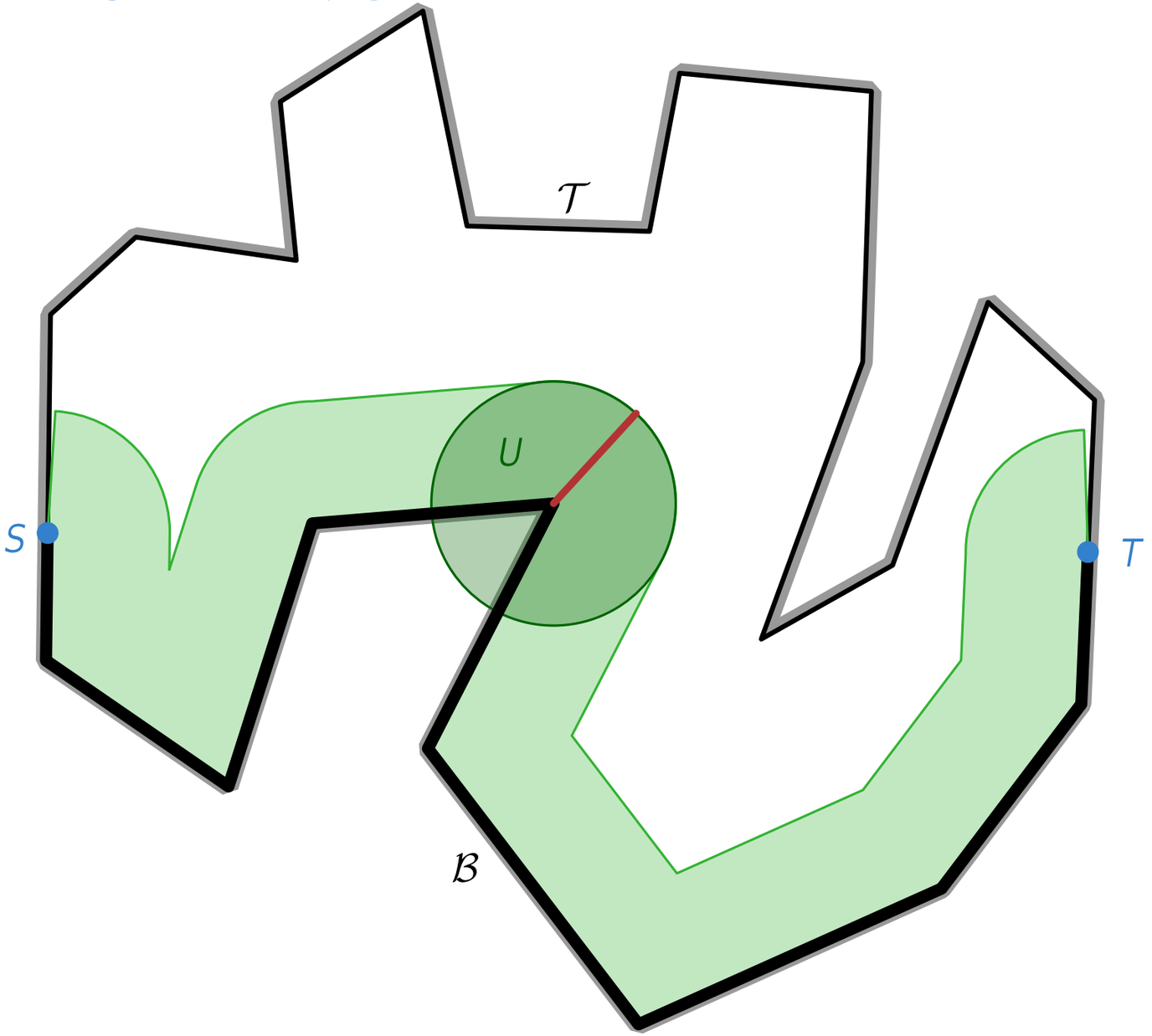
Test for complete blockage

Compute $\mathcal{D} = \mathcal{B} \oplus U$

Test if $\mathcal{T}$ intersects $\mathcal{D}$

Maximize detour

# Simple Polygon, 1 Barrier




Ingredients:

Test for complete blockage

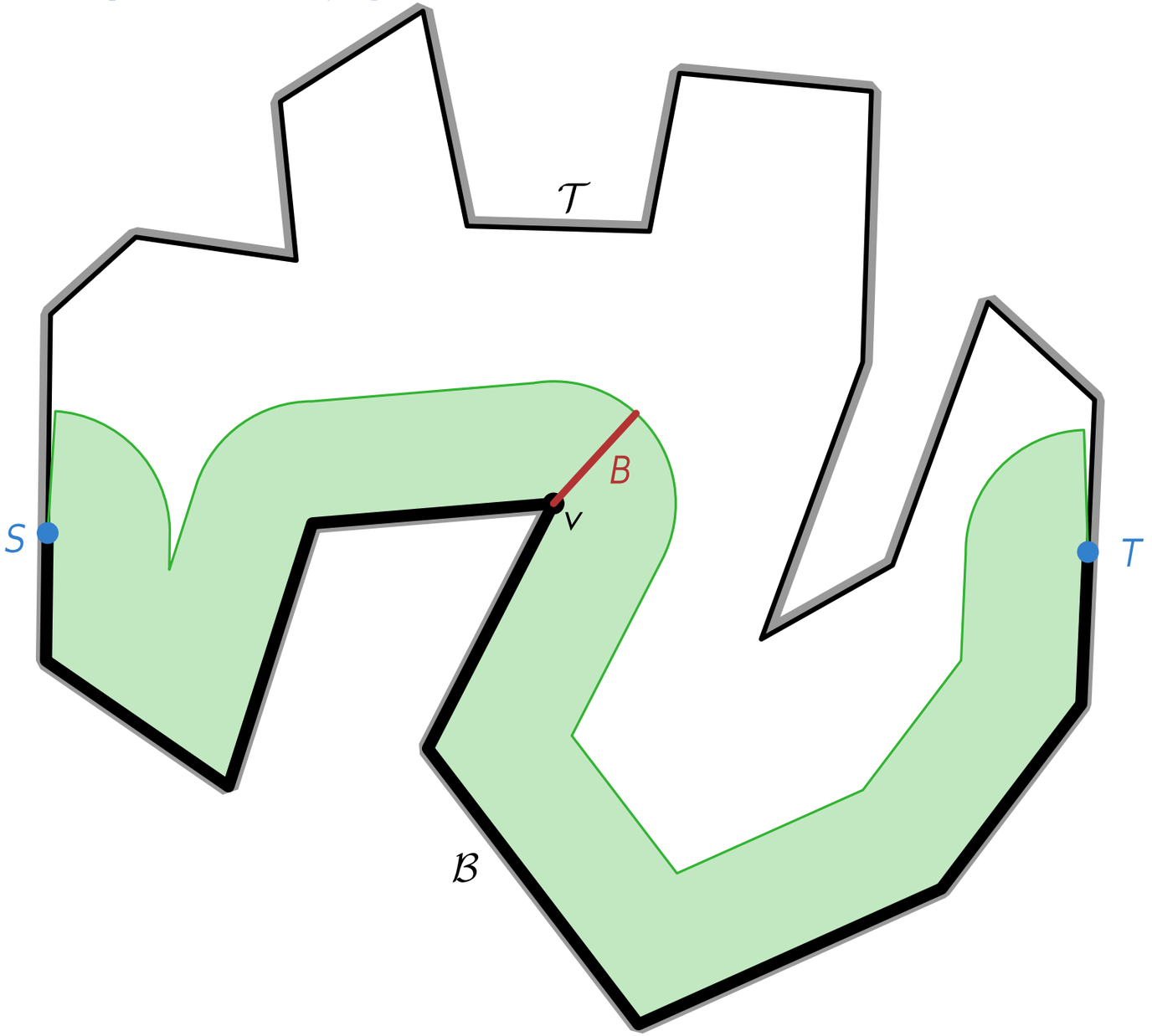
Compute $\mathcal{D} = \mathcal{B} \oplus U$

Test if $\mathcal{T}$ intersects $\mathcal{D}$

$\Longrightarrow$ $O(n)$ time

Maximize detour

# Simple Polygon, 1 Barrier




Ingredients:

Test for complete blockage

Compute $\mathcal{D} = \mathcal{B} \oplus U$

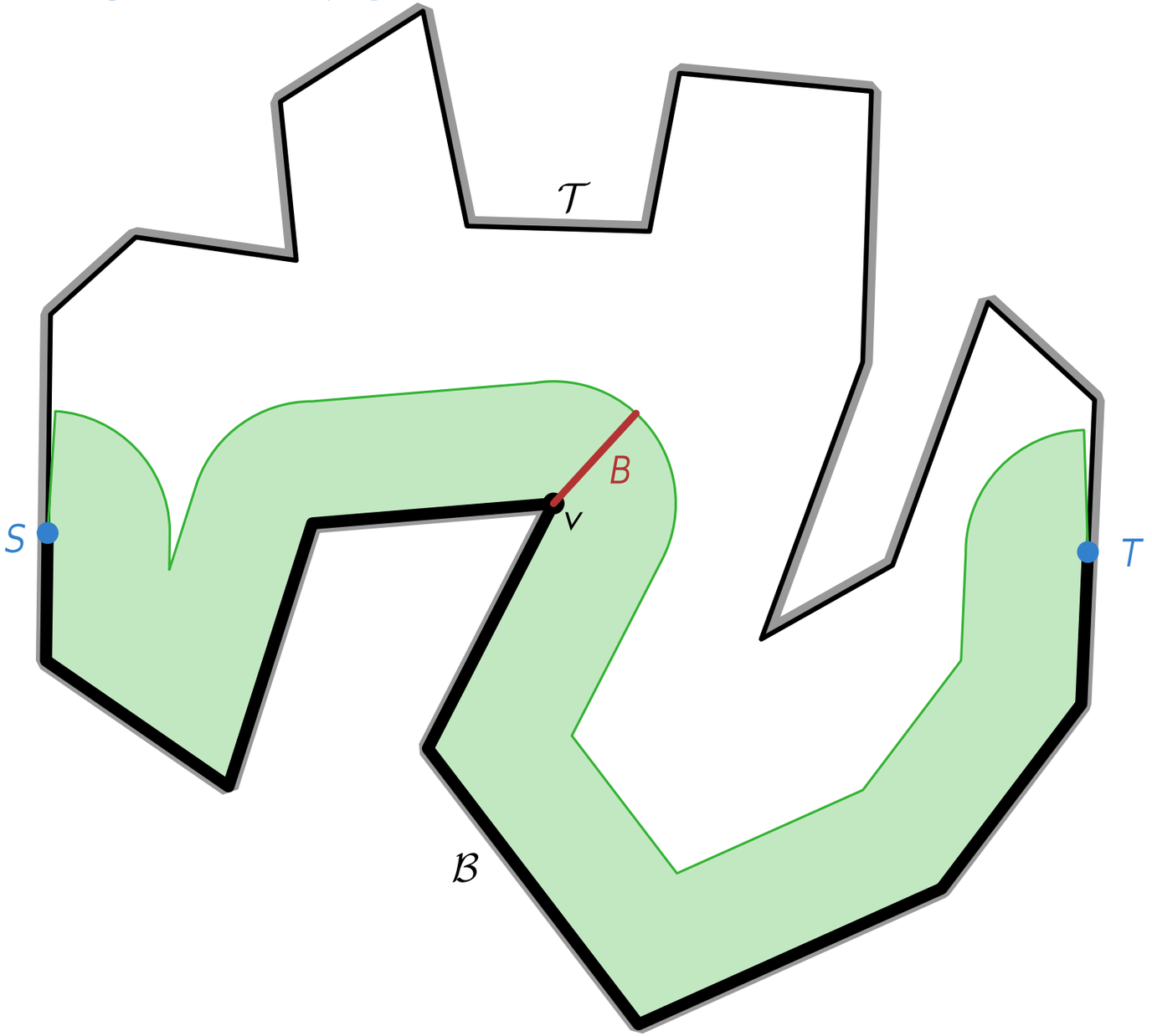
Test if $\mathcal{T}$ intersects $\mathcal{D}$

$\Longrightarrow$ $O(n)$ time

Maximize detour

**Lem.** $\exists$ opt barrier $B = \overline{bv}$ with endpoint at a vertex $v$

# Simple Polygon, 1 Barrier




Ingredients:

Test for complete blockage

Compute $\mathcal{D} = \mathcal{B} \oplus U$

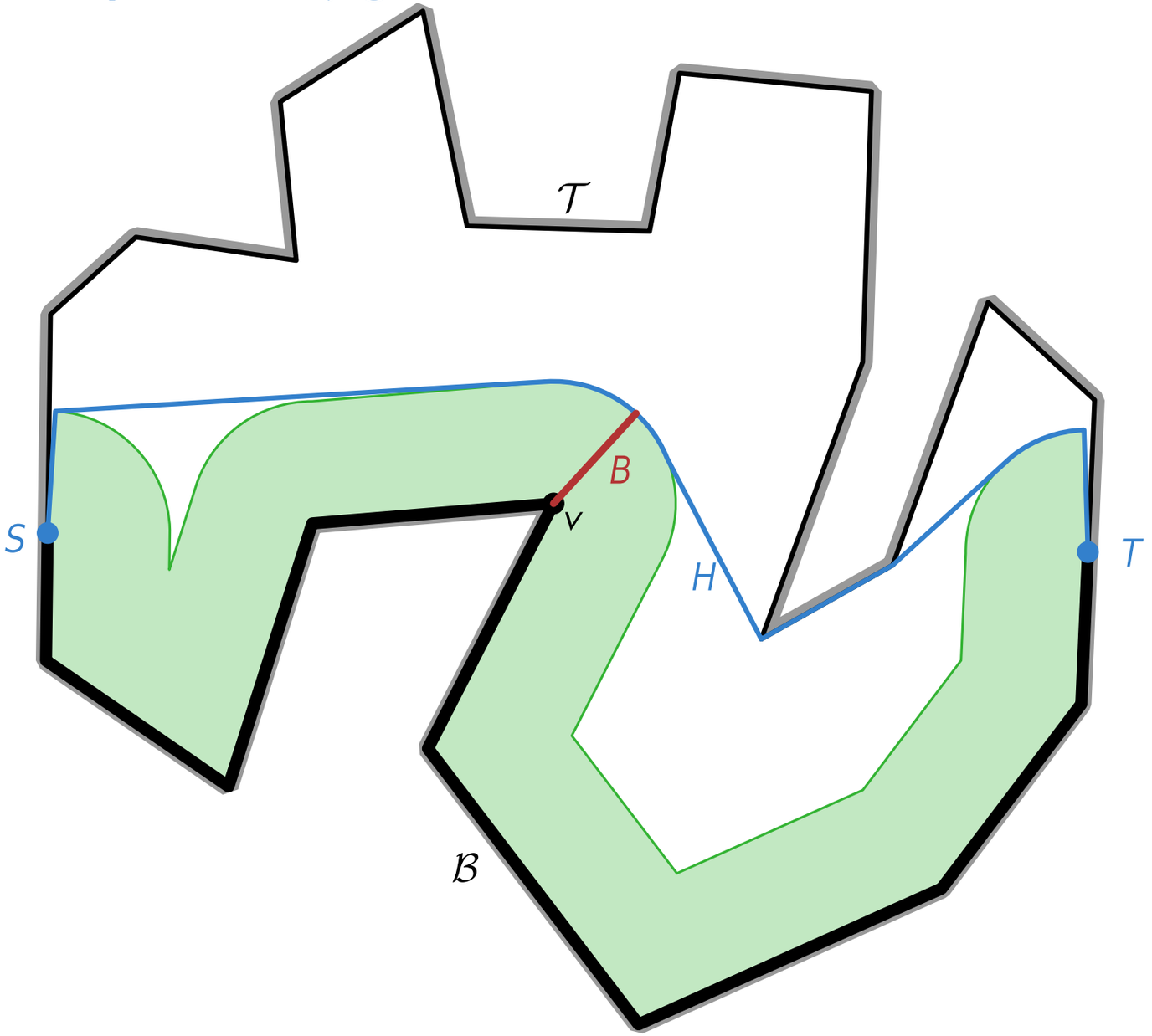
Test if $\mathcal{T}$ intersects $\mathcal{D}$

$\Longrightarrow$ $O(n)$ time

Maximize detour

**Lem.** $\exists$ opt barrier $B = \overline{bv}$ with endpoint at a vertex $v$ and $b$ on $\partial\mathcal{D}$

# Simple Polygon, 1 Barrier




Ingredients:

Test for complete blockage

Compute $\mathcal{D} = \mathcal{B} \oplus U$

Test if $\mathcal{T}$ intersects $\mathcal{D}$

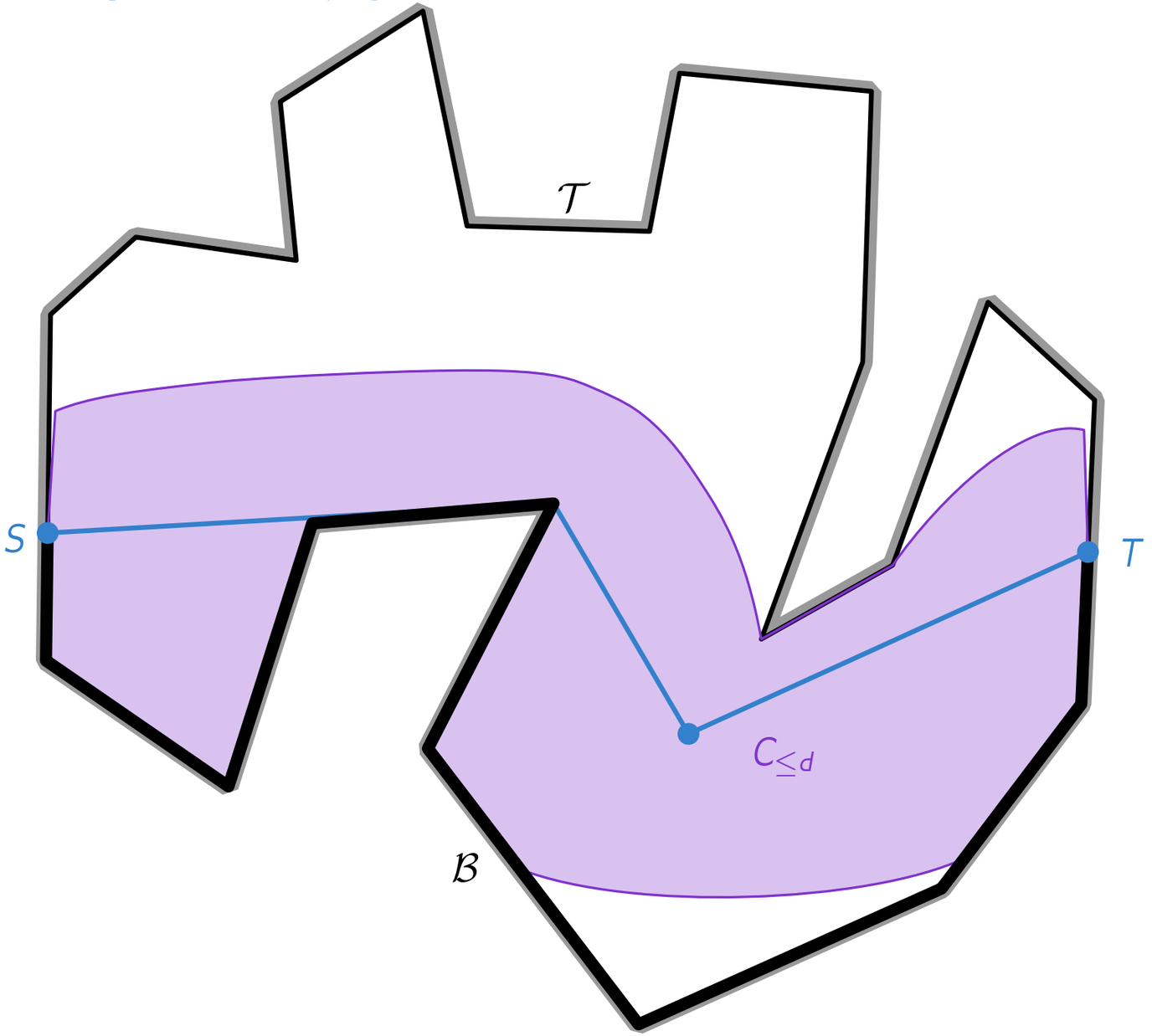
$\Longrightarrow$ $O(n)$ time

Maximize detour

**Lem.** $\exists$ opt barrier $B = \overline{bv}$ with endpoint at a vertex $v$ and $b$ on $\partial\mathcal{D} \cap H$

$H$ shortest $s, t$-path in $P \setminus \mathcal{D}$

# Simple Polygon, 1 Barrier




Ingredients:

Test for complete blockage

Compute $\mathcal{D} = \mathcal{B} \oplus U$

Test if $\mathcal{T}$ intersects $\mathcal{D}$

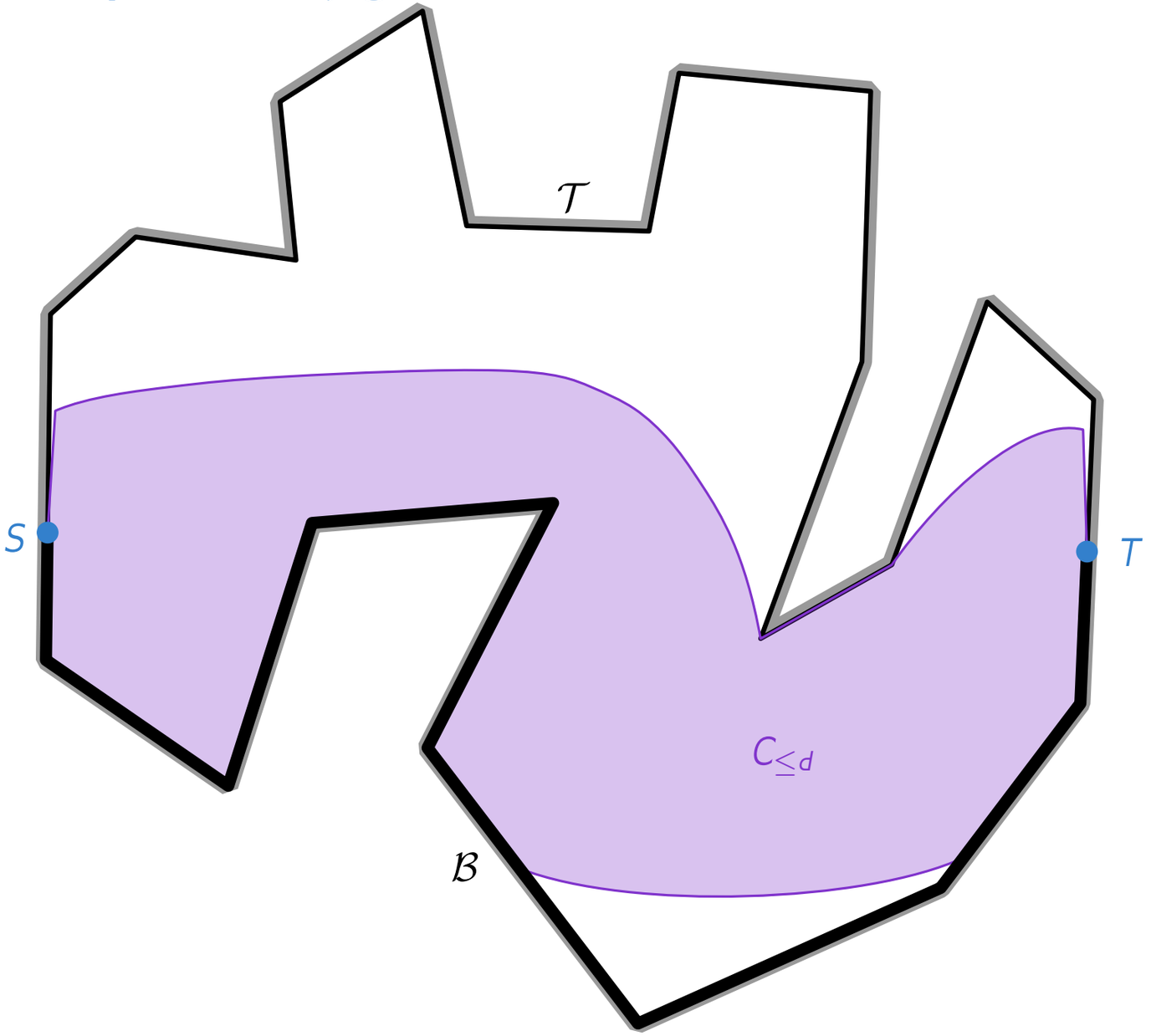
$\Longrightarrow$ $O(n)$ time

Maximize detour

**Lem.** $\exists$ opt barrier $B = \overline{bv}$ with endpoint at a vertex $v$ and $b$ on $\partial\mathcal{D} \cap H$

$H$ shortest $s, t$-path in $P \setminus \mathcal{D}$

# Simple Polygon, 1 Barrier




Ingredients:

Test for complete blockage

Compute $\mathcal{D} = \mathcal{B} \oplus U$

Test if $\mathcal{T}$ intersects $\mathcal{D}$

$\Longrightarrow$ $O(n)$ time

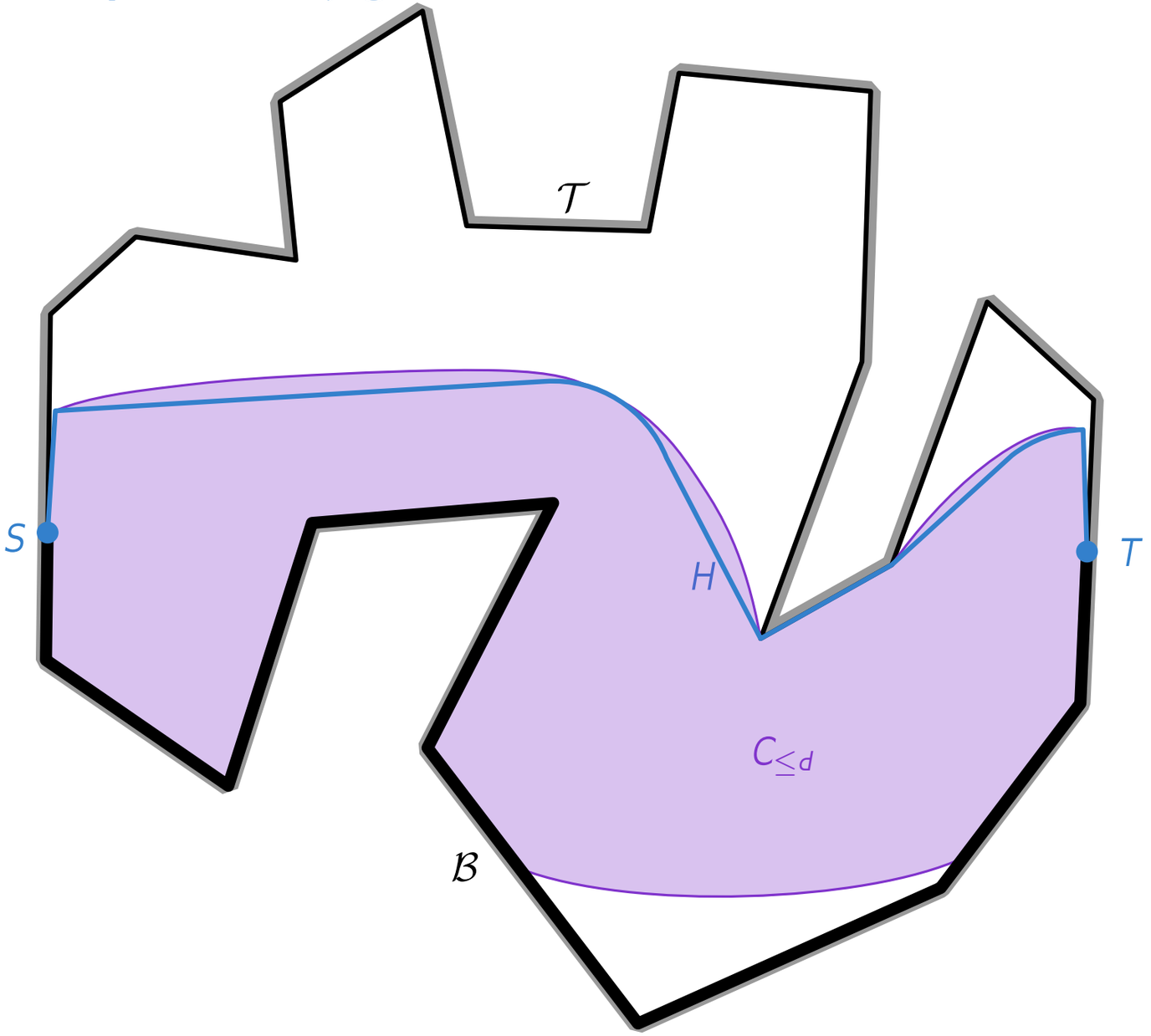
Maximize detour

**Lem.** $\exists$ opt barrier $B = \overline{bv}$ with endpoint at a vertex $v$ and $b$ on $\partial\mathcal{D} \cap H$

$H$ shortest $s, t$-path in $P \setminus \mathcal{D}$

**Lem.** $C_{\leq d}$ is geodesically convex

# Simple Polygon, 1 Barrier




Ingredients:

Test for complete blockage

Compute $\mathcal{D} = \mathcal{B} \oplus U$

Test if $\mathcal{T}$ intersects $\mathcal{D}$

$\Longrightarrow$ $O(n)$ time

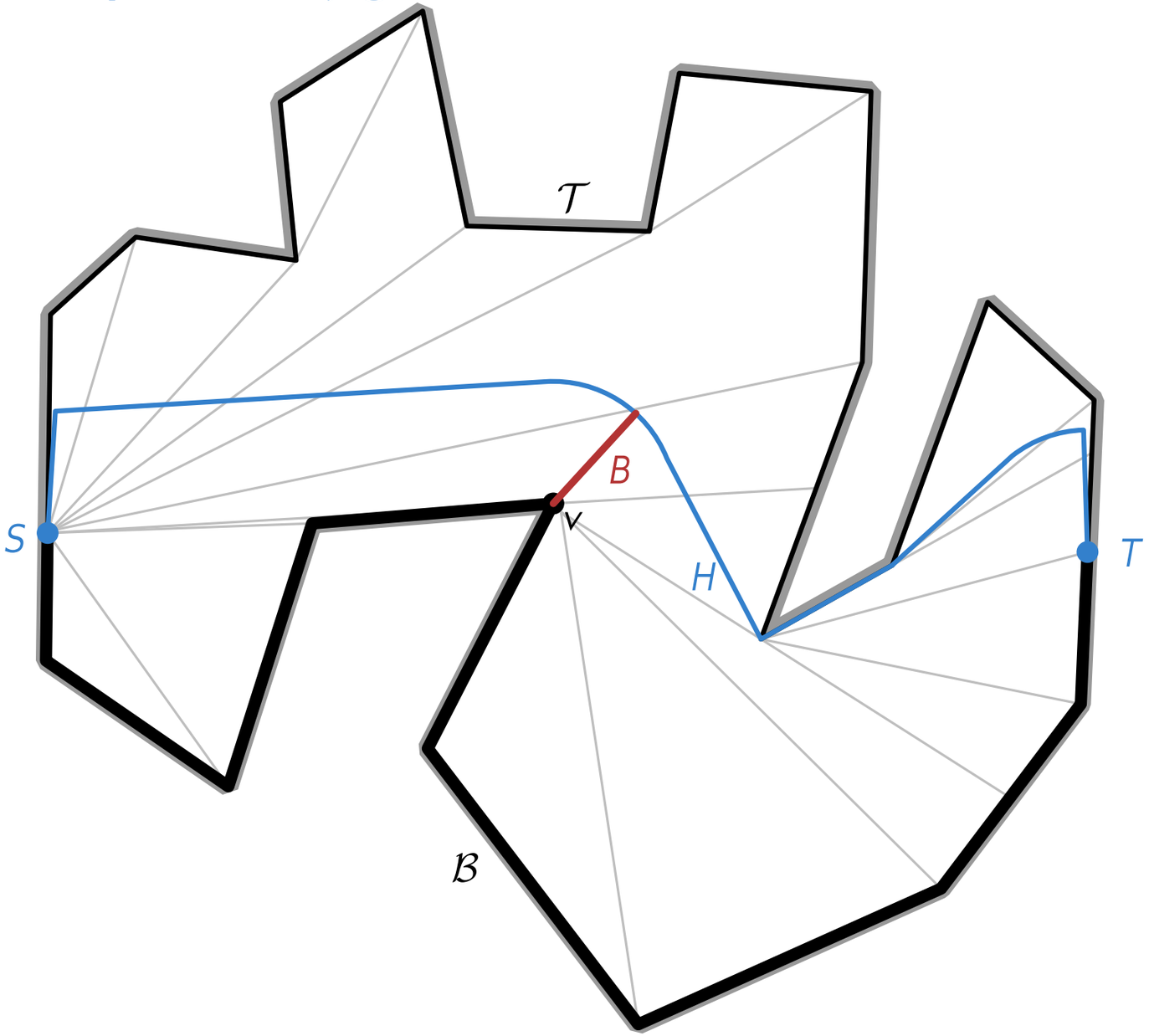
Maximize detour

**Lem.** $\exists$ opt barrier $B = \overline{bv}$ with endpoint at a vertex $v$ and $b$ on $\partial\mathcal{D} \cap H$

$H$ shortest $s, t$-path in $P \setminus \mathcal{D}$

**Lem.** $C_{\leq d}$ is geodesically convex

# Simple Polygon, 1 Barrier




Ingredients:

Test for complete blockage

Compute $\mathcal{D} = \mathcal{B} \oplus U$

Test if $\mathcal{T}$ intersects $\mathcal{D}$

$\Longrightarrow$ $O(n)$ time

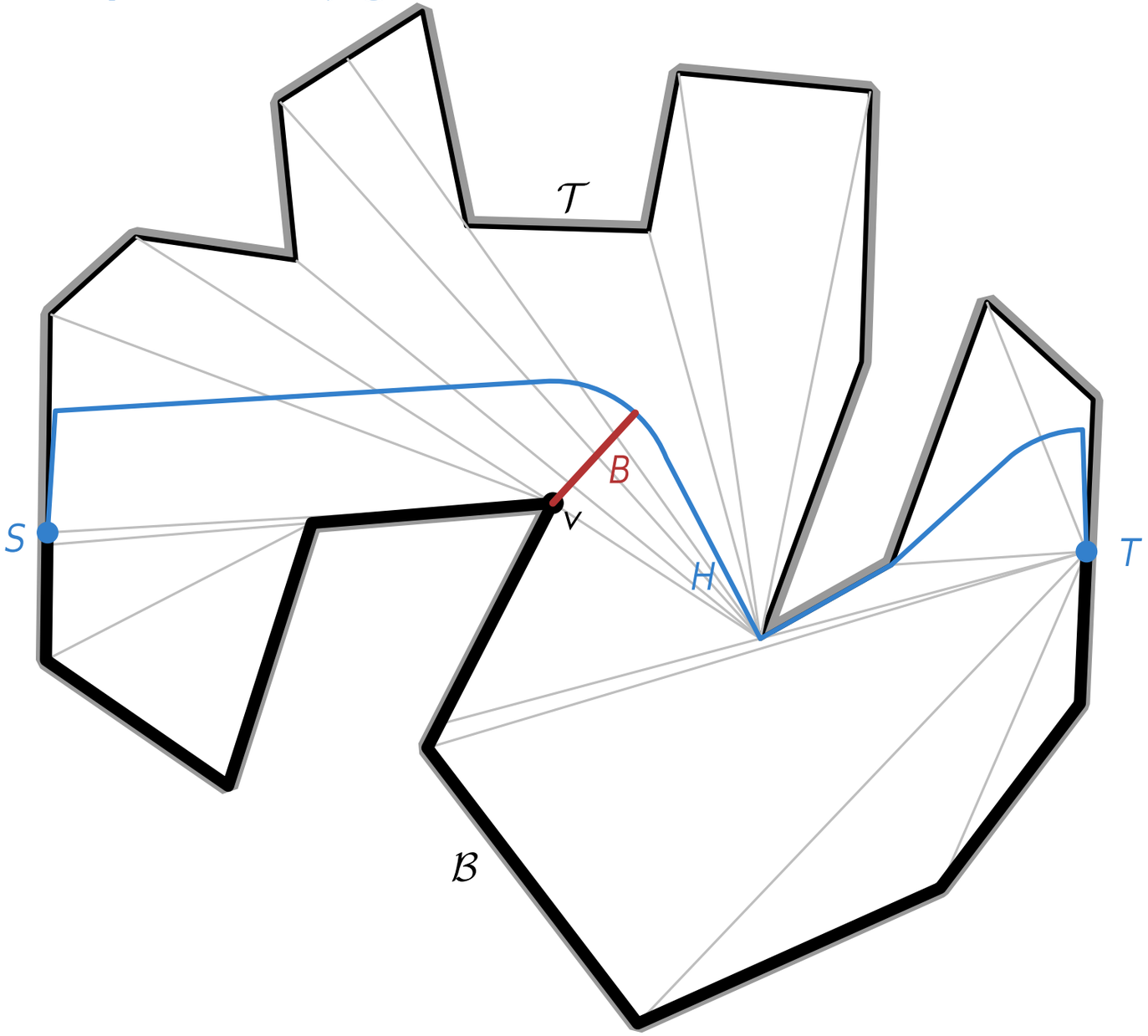
Maximize detour

**Lem.** $\exists$ opt barrier $B = \overline{bv}$ with endpoint at a vertex $v$ and $b$ on $\partial\mathcal{D} \cap H$

$H$ shortest $s, t$-path in $P \setminus \mathcal{D}$

$H$ intersects SPM($s$) and SPM($t$) only $O(n)$ times.

# Simple Polygon, 1 Barrier




Ingredients:

Test for complete blockage

Compute $\mathcal{D} = \mathcal{B} \oplus U$

Test if $\mathcal{T}$ intersects $\mathcal{D}$

$\Longrightarrow$ $O(n)$ time

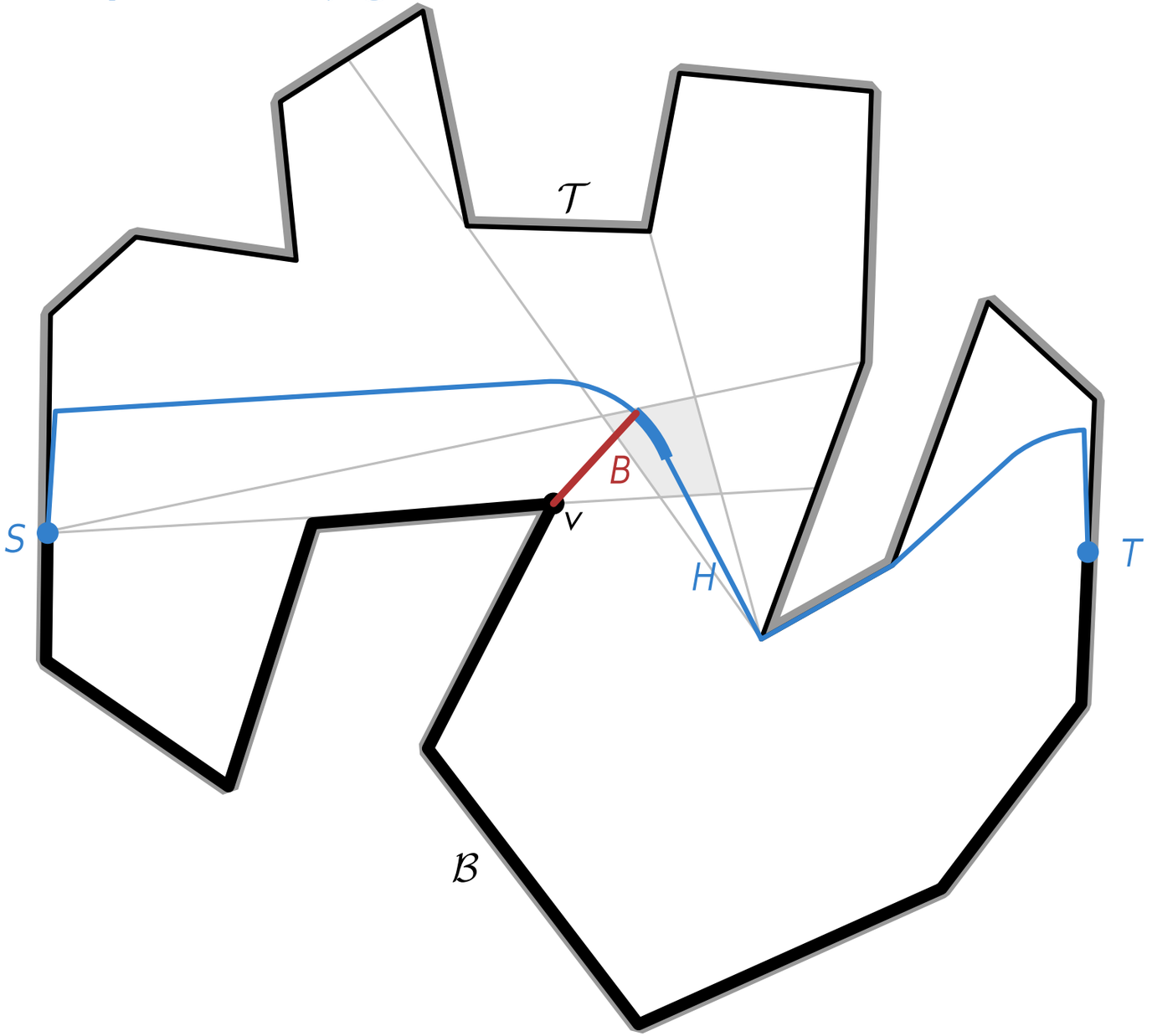
Maximize detour

**Lem.** $\exists$ opt barrier $B = \overline{bv}$ with endpoint at a vertex $v$ and $b$ on $\partial\mathcal{D} \cap H$

$H$ shortest $s, t$-path in $P \setminus \mathcal{D}$

$H$ intersects SPM($s$) and SPM($t$) only $O(n)$ times.

# Simple Polygon, 1 Barrier




Ingredients:

Test for complete blockage

Compute $\mathcal{D} = \mathcal{B} \oplus U$

Test if $\mathcal{T}$ intersects $\mathcal{D}$

$\Longrightarrow$ $O(n)$ time

Maximize detour

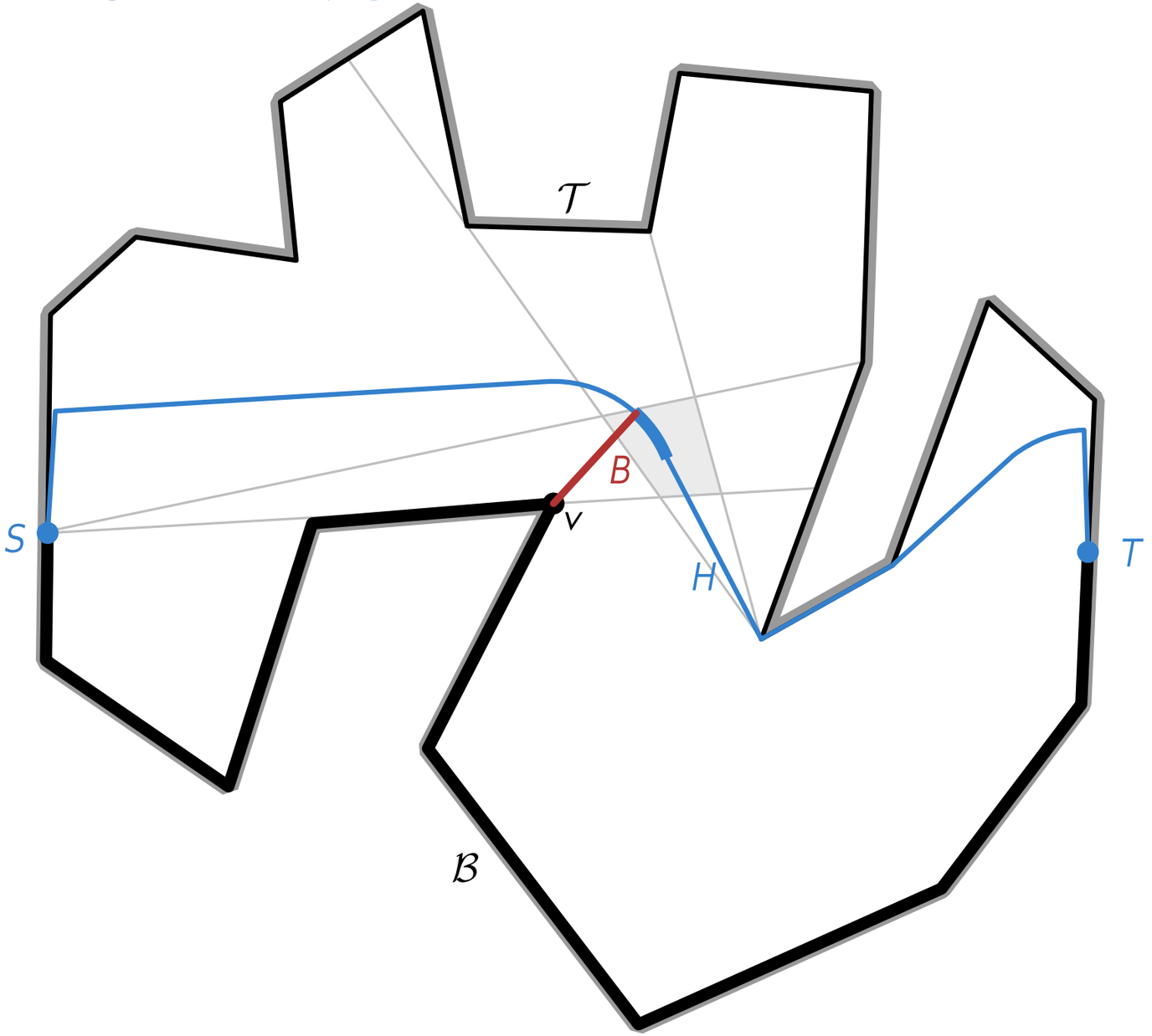
**Lem.** $\exists$ opt barrier $B = \overline{bv}$ with endpoint at a vertex $v$ and $b$ on $\partial\mathcal{D} \cap H$

$H$ shortest $s, t$-path in $P \setminus \mathcal{D}$

$H$ intersects SPM($s$) and SPM($t$) only $O(n)$ times.

compute opt barrier for each piece in $O(1)$ time

# Simple Polygon, 1 Barrier




Ingredients:

Test for complete blockage

Compute $\mathcal{D} = \mathcal{B} \oplus U$

Test if $\mathcal{T}$ intersects $\mathcal{D}$

$\Longrightarrow$ $O(n)$ time

Maximize detour

**Lem.** $\exists$ opt barrier $B = \overline{bv}$ with endpoint at a vertex $v$ and $b$ on $\partial\mathcal{D} \cap H$

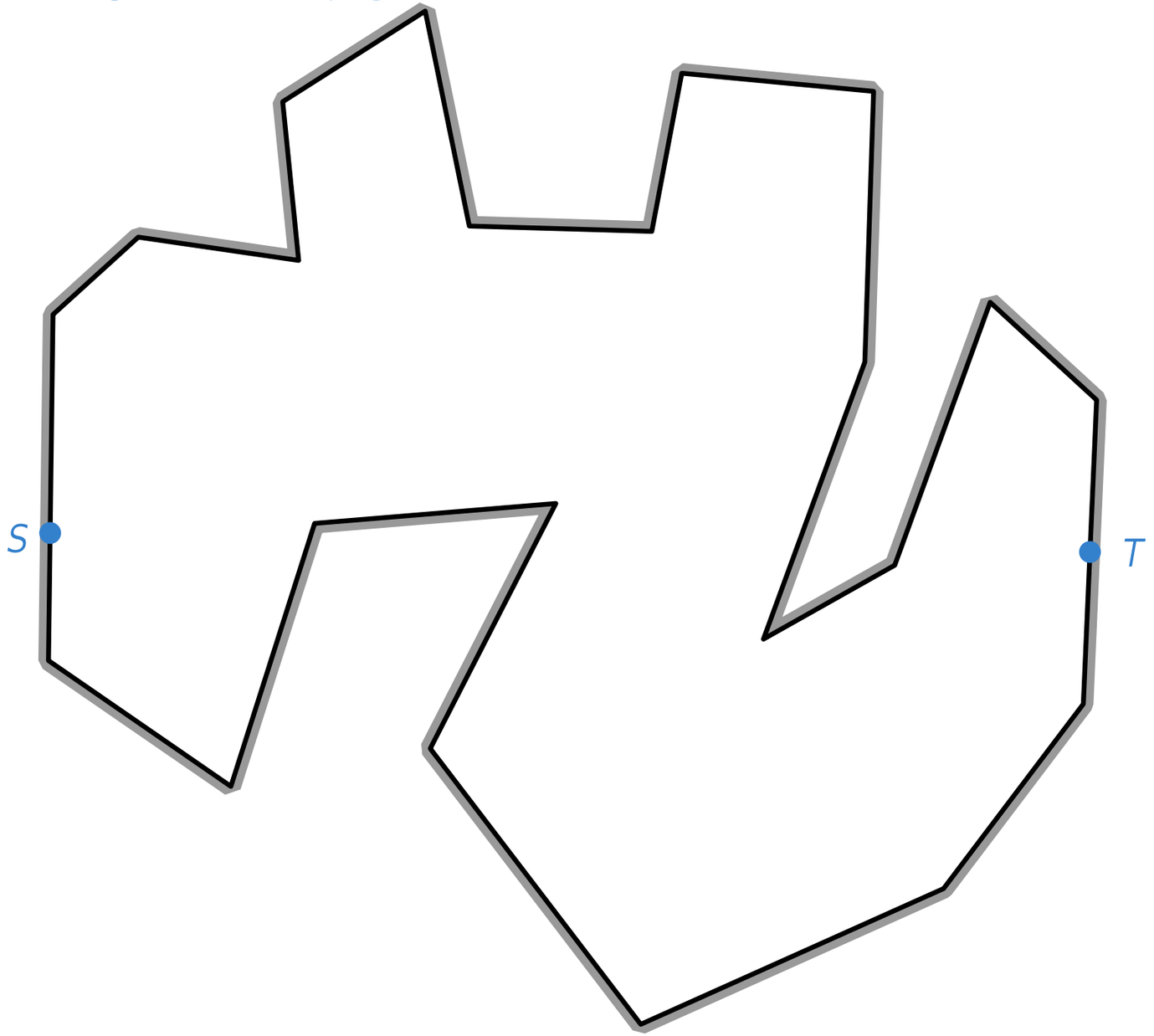
$H$ shortest $s, t$-path in $P \setminus \mathcal{D}$

$H$ intersects SPM($s$) and SPM($t$) only $O(n)$ times.

compute opt barrier for each piece in $O(1)$ time

**Thm.** Compute opt $B$ in $O(n)$ time.

# Simple Polygon, >1 Barrier

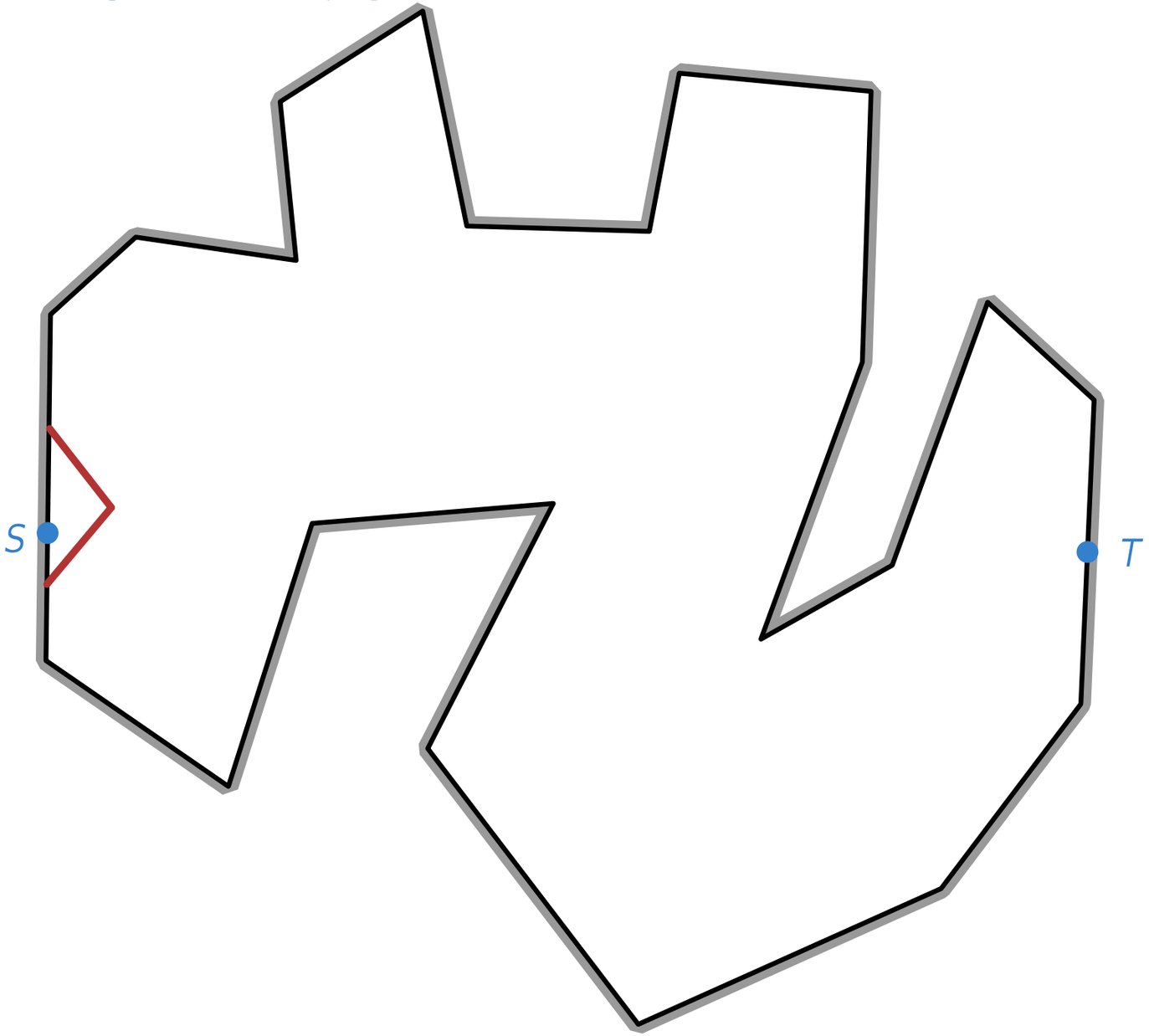



Ingredients:

Test for complete blockage

Maximize detour

# Simple Polygon, >1 Barrier

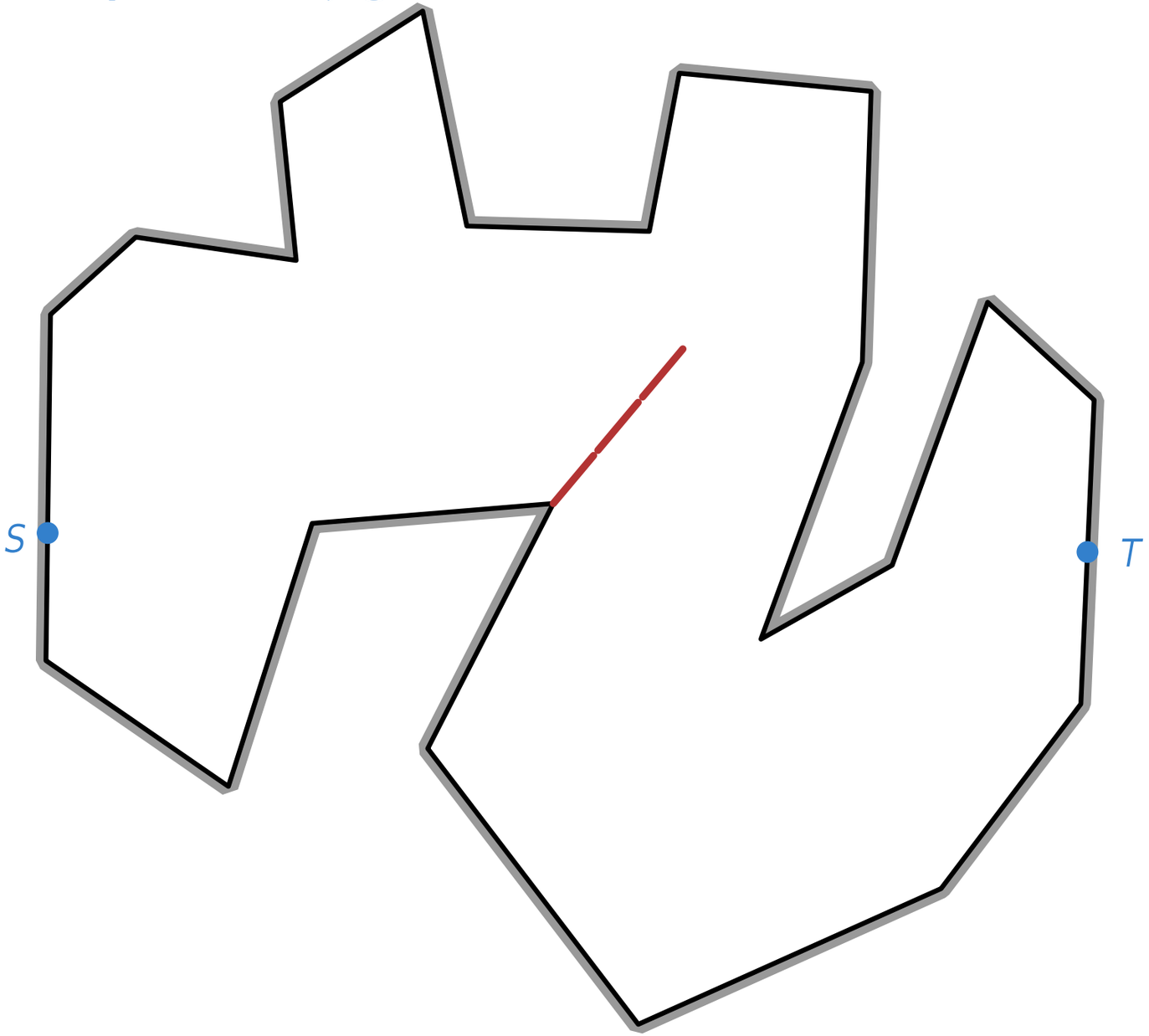



Ingredients:

Test for complete blockage

Maximize detour

# Simple Polygon, >1 Barrier




Ingredients:

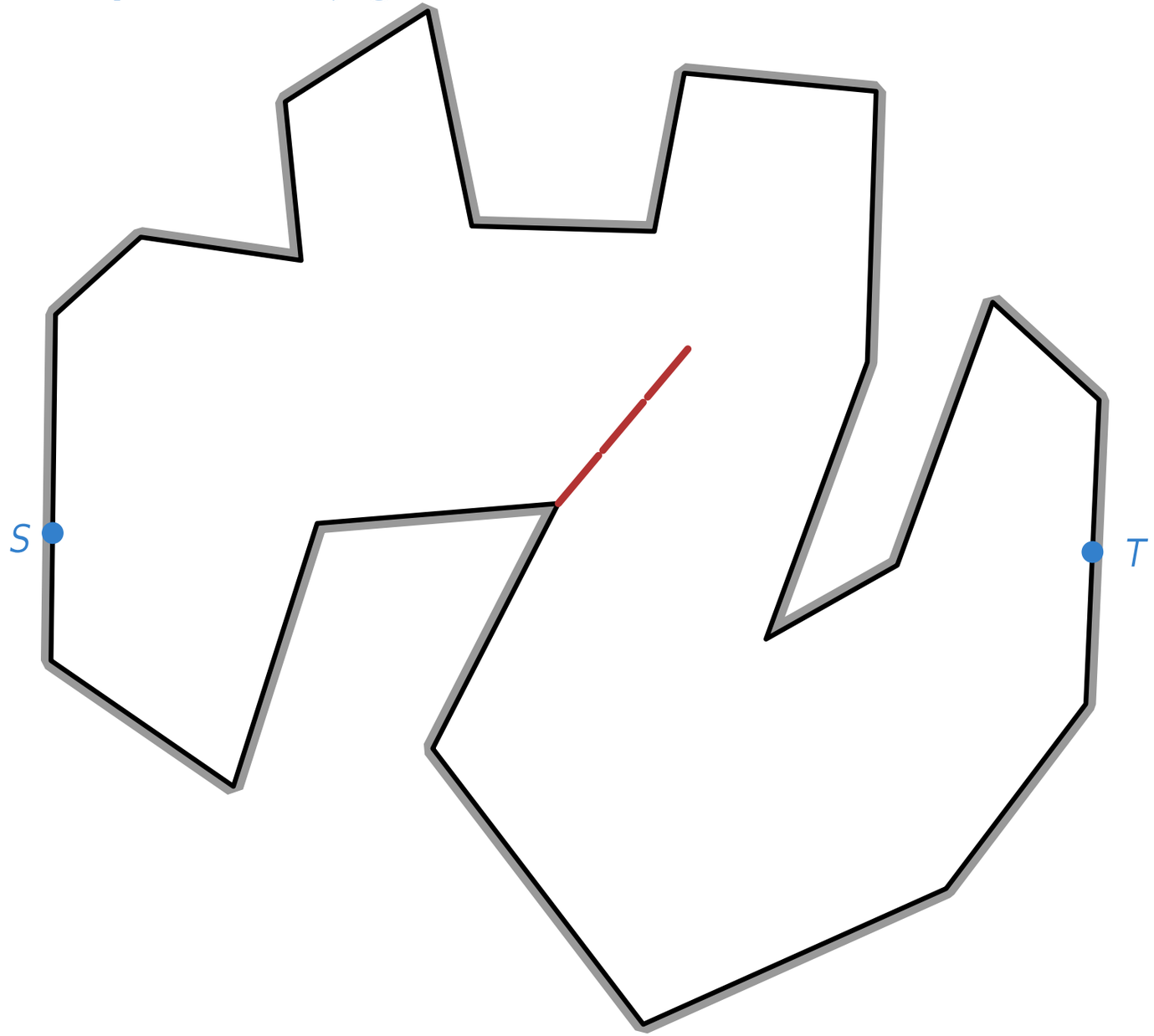
Test for complete blockage

Maximize detour

No complete blockage $\Longrightarrow$

**Lem.** Opt placement glues all barriers into a super barrier *B*.

# Simple Polygon, >1 Barrier




Ingredients:

Test for complete blockage

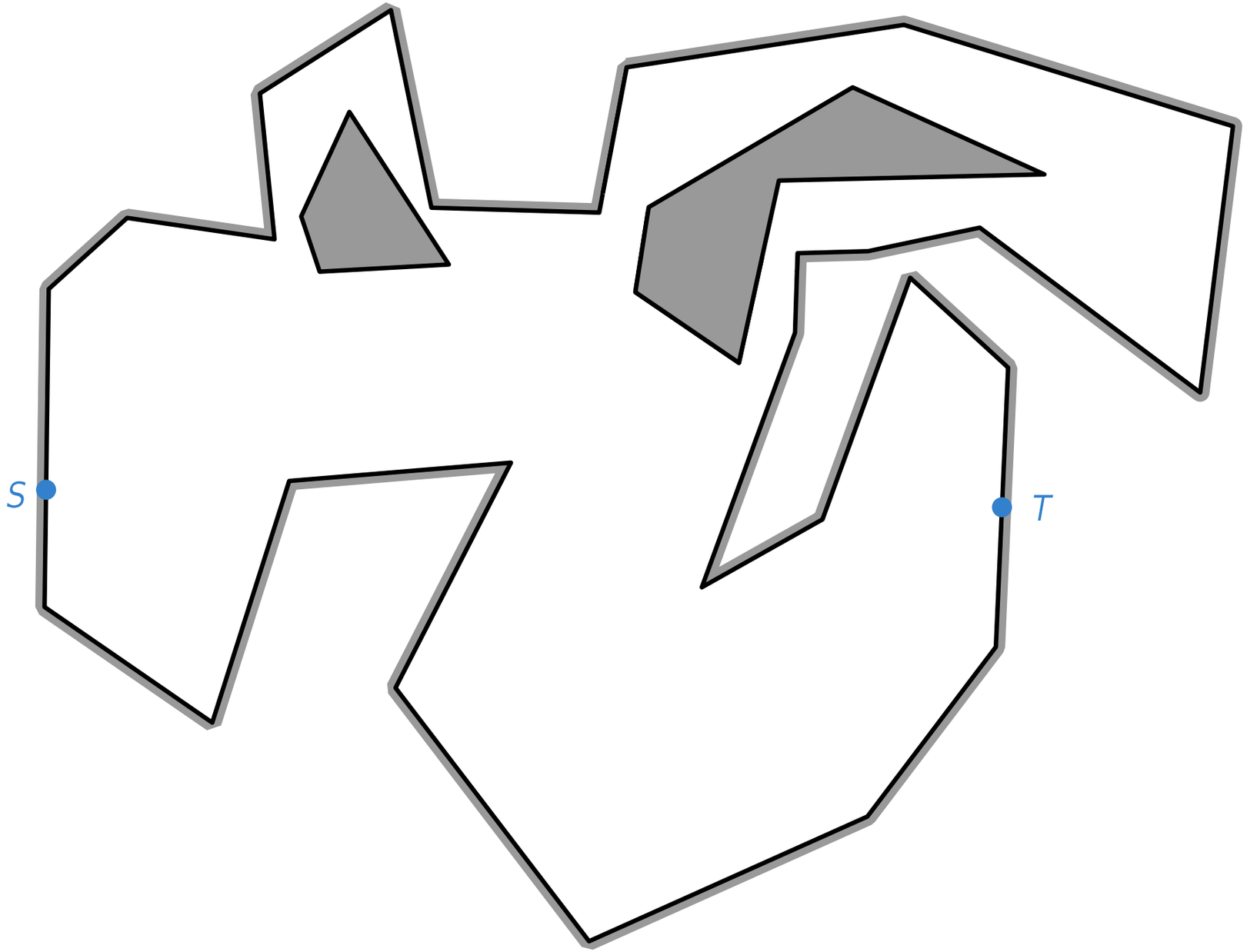
Maximize detour

No complete blockage $\Longrightarrow$

**Lem.** Opt placement glues all barriers into a super barrier $B$.

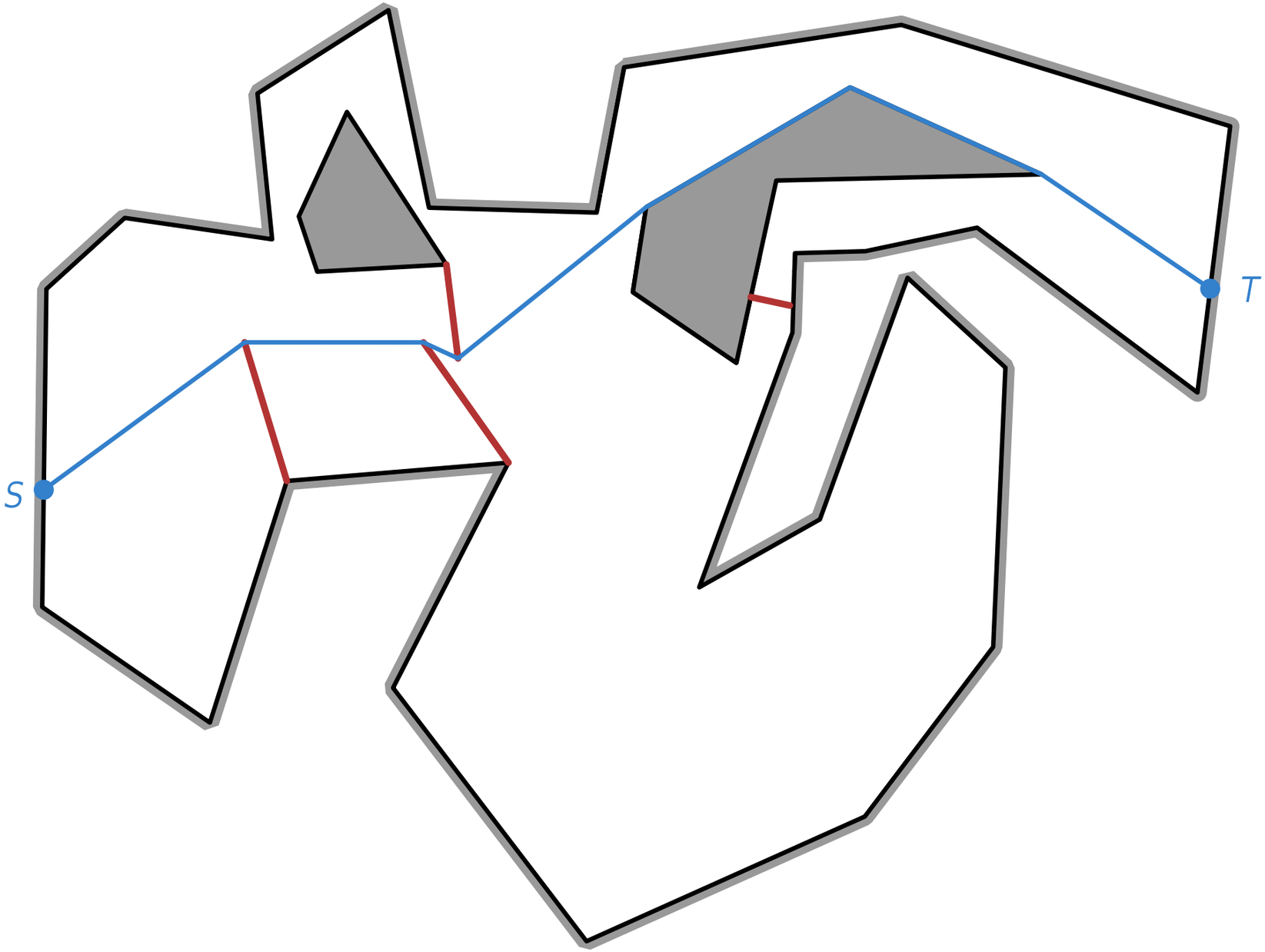
**Thm.** Compute opt $B$ in $O(n)$ time.

# Future Work




O(1) holes, many unit barriers

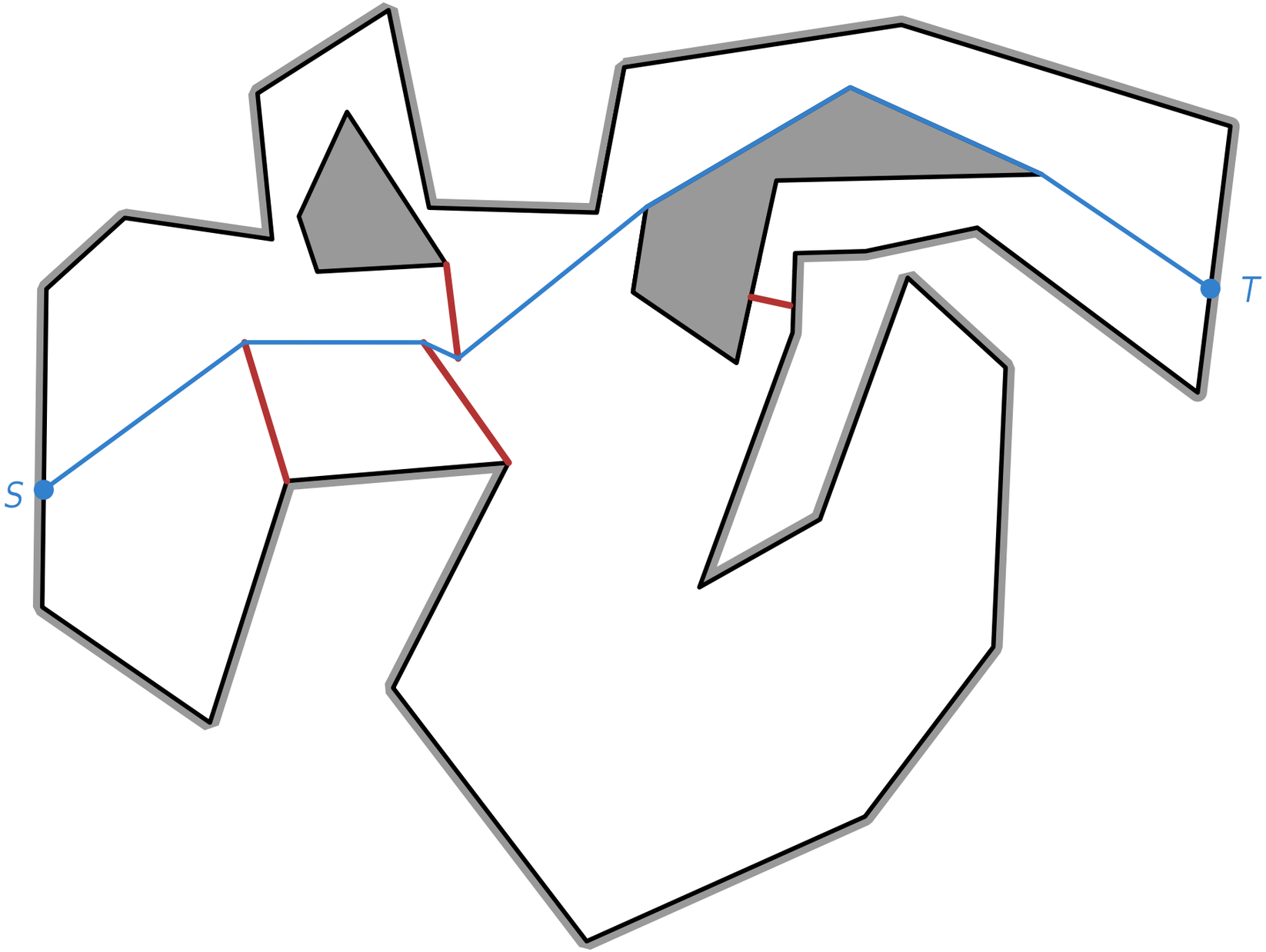
# Future Work




O(1) holes, many unit barriers

Variant with total budget

# Future Work




O(1) holes, many unit barriers

Variant with total budget

Thank you!