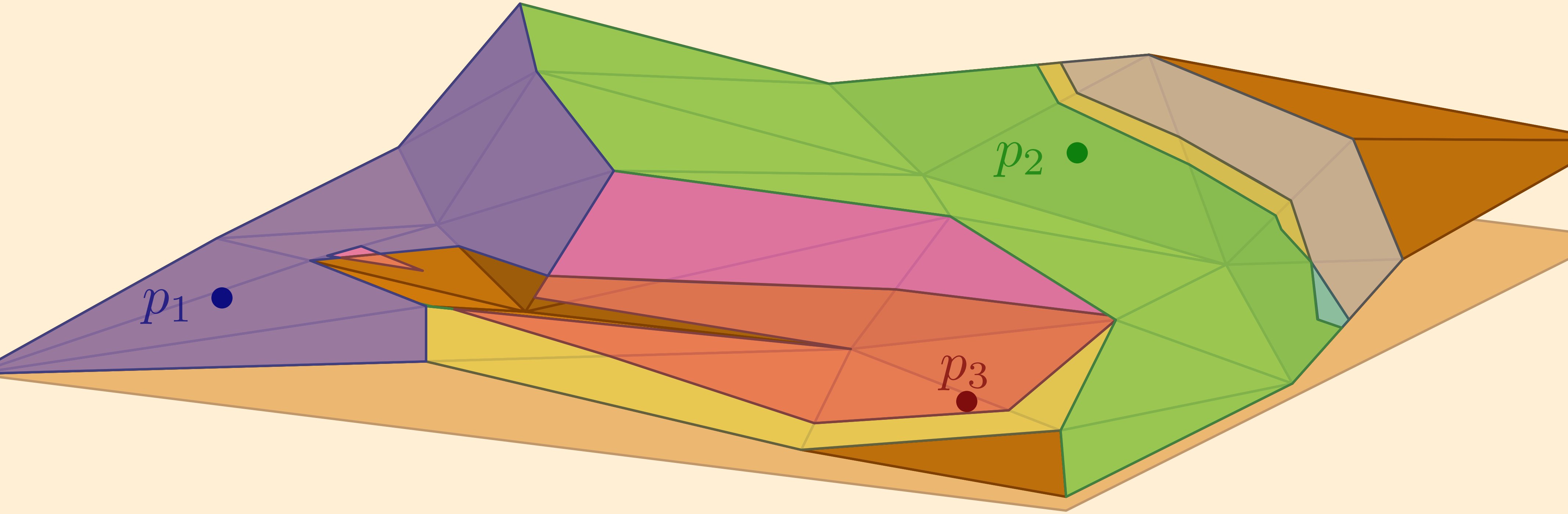
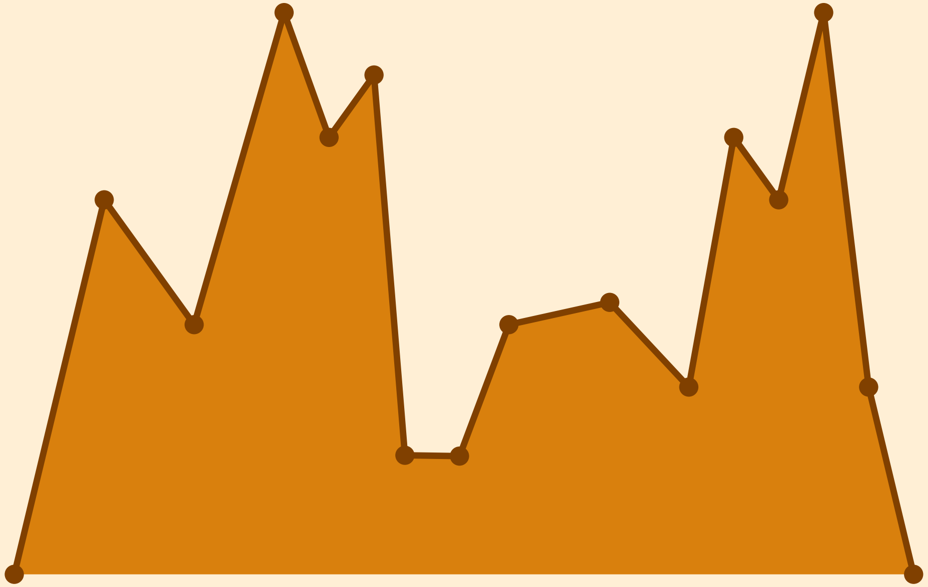
Terrain Visibility with Multiple Viewpoints

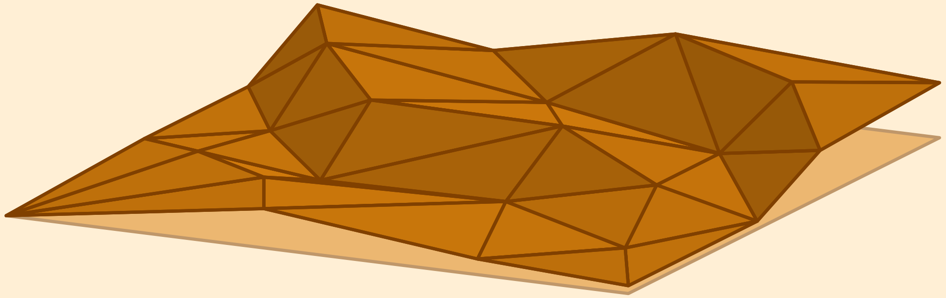
Ferran Hurtado
Maarten Löffler
Inês Matos
Vera Sacristán
Maria Saumell
Rodrigo I. Silveira
Frank Staals



Terrain Visibility with Multiple Viewpoints



1.5D Terrain $\mathcal{T}$

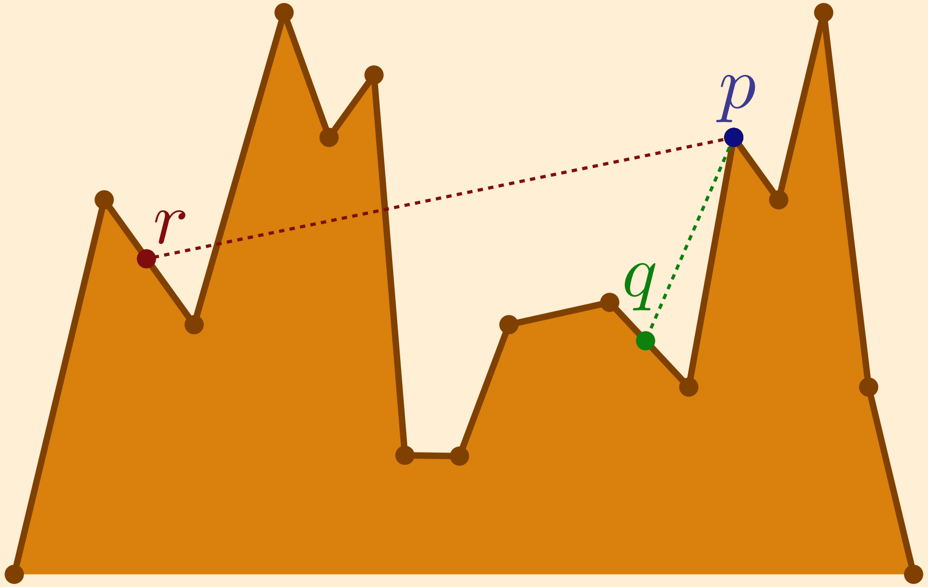


2.5D Terrain $\mathcal{T}$

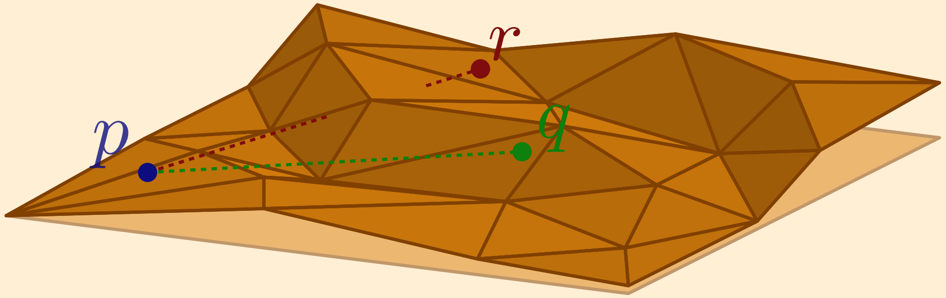
Terrain Visibility with Multiple Viewpoints

q visible from viewpoint p

r not visible from viewpoint p



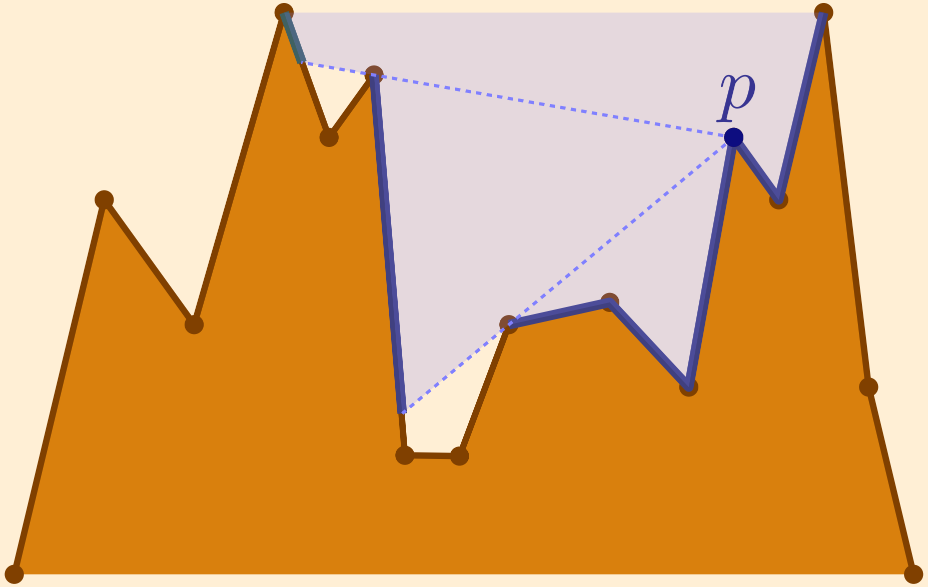
1.5D Terrain $\mathcal{T}$



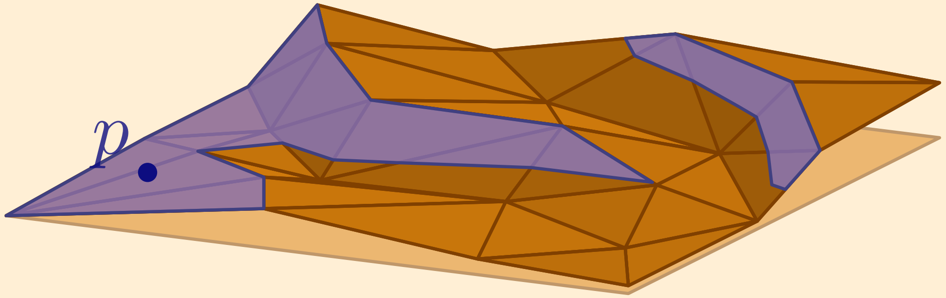
2.5D Terrain $\mathcal{T}$

Terrain Visibility with Multiple Viewpoints

The viewshed $\mathcal{V}_{\mathcal{T}}(p)$ is the set of points visible from p.

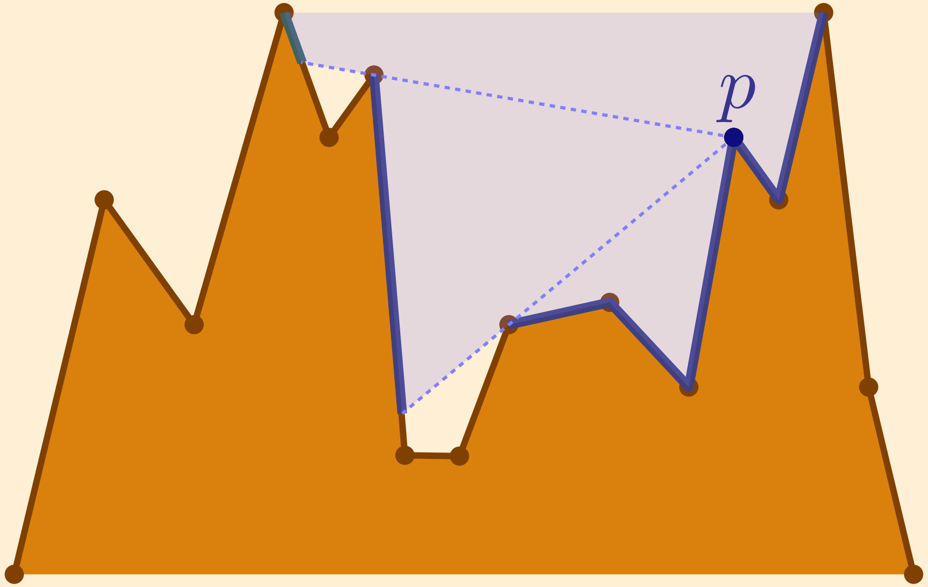


1.5D Terrain $\mathcal{T}$

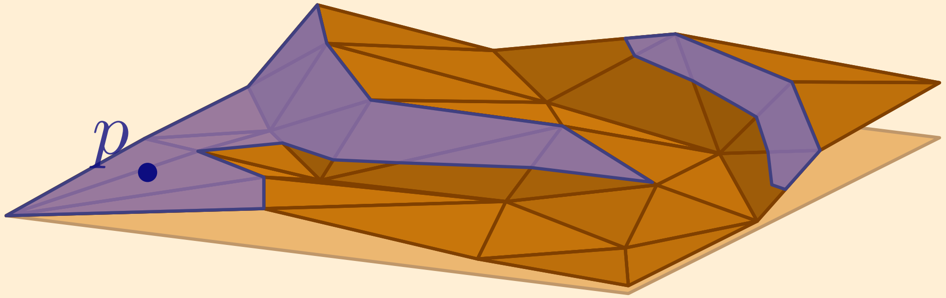


2.5D Terrain $\mathcal{T}$

Terrain Visibility with Multiple Viewpoints

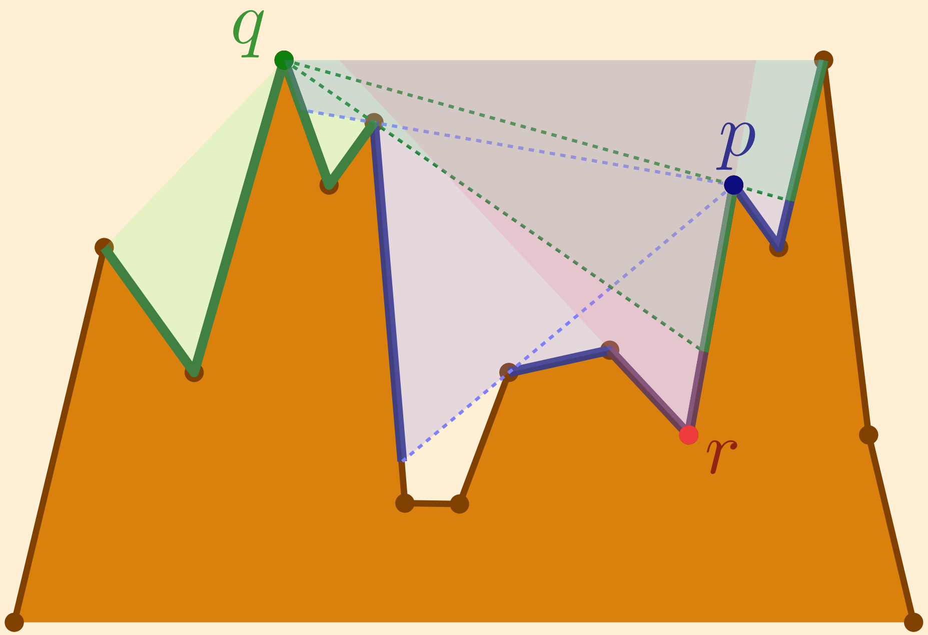


1.5D Terrain $\mathcal{T}$

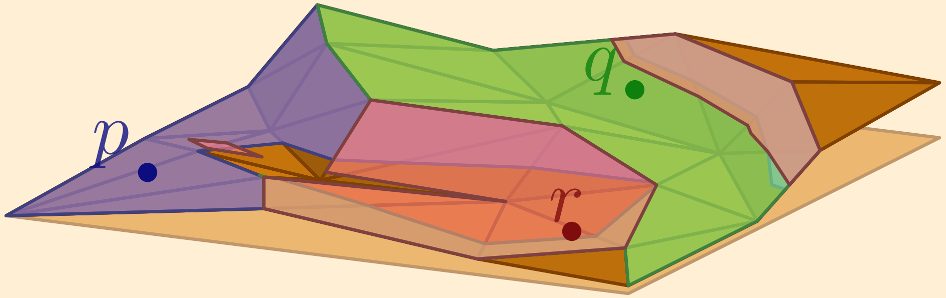


2.5D Terrain $\mathcal{T}$

Terrain Visibility with Multiple Viewpoints



1.5D Terrain $\mathcal{T}$

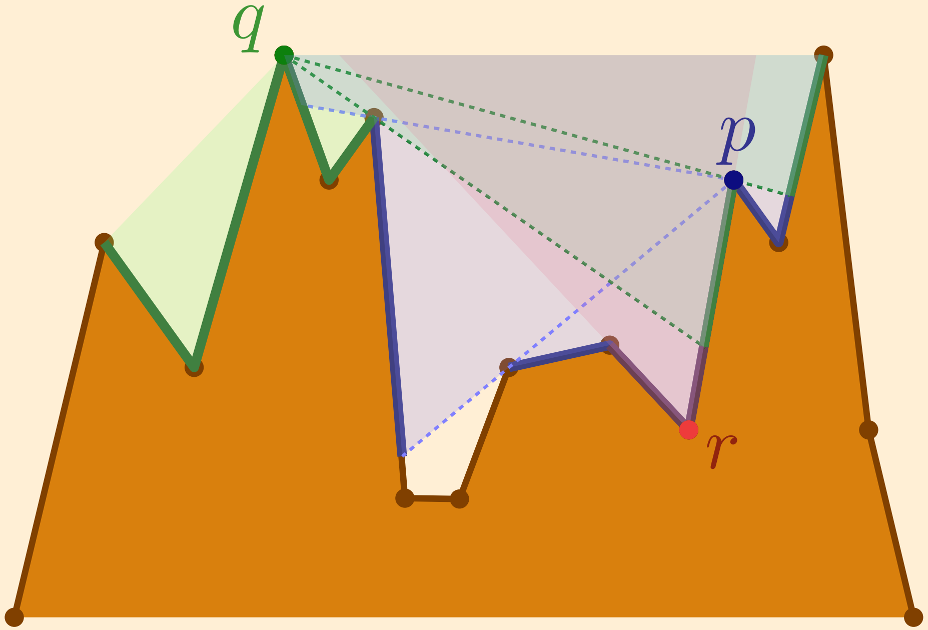


2.5D Terrain $\mathcal{T}$

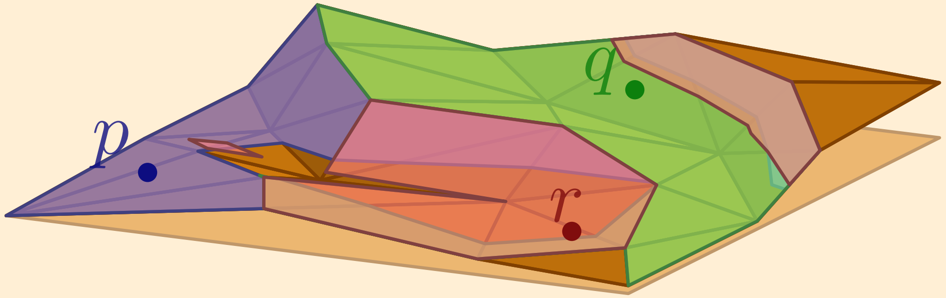
Terrain Visibility with Multiple Viewpoints

Given a set $\mathcal{P}$ of viewpoints

Is a query point s visible?



1.5D Terrain $\mathcal{T}$



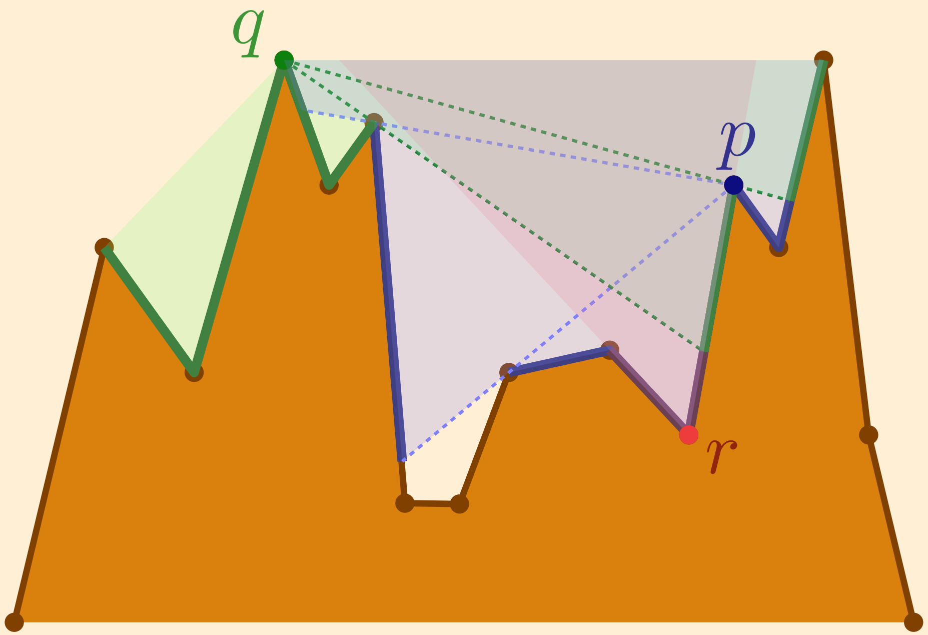
2.5D Terrain $\mathcal{T}$

Terrain Visibility with Multiple Viewpoints

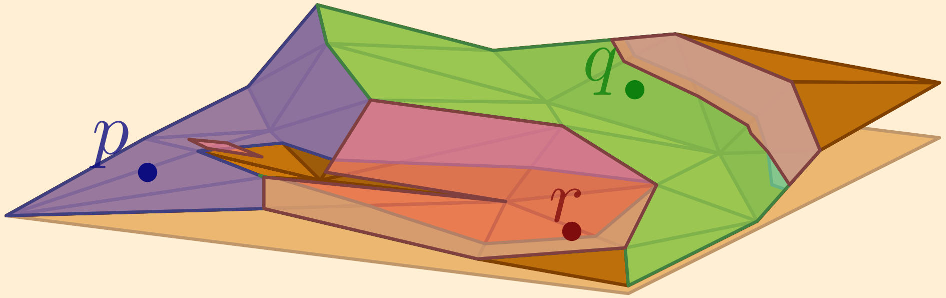
Given a set $\mathcal{P}$ of viewpoints

Is a query point s visible?

Which viewpoints see s?



1.5D Terrain $\mathcal{T}$

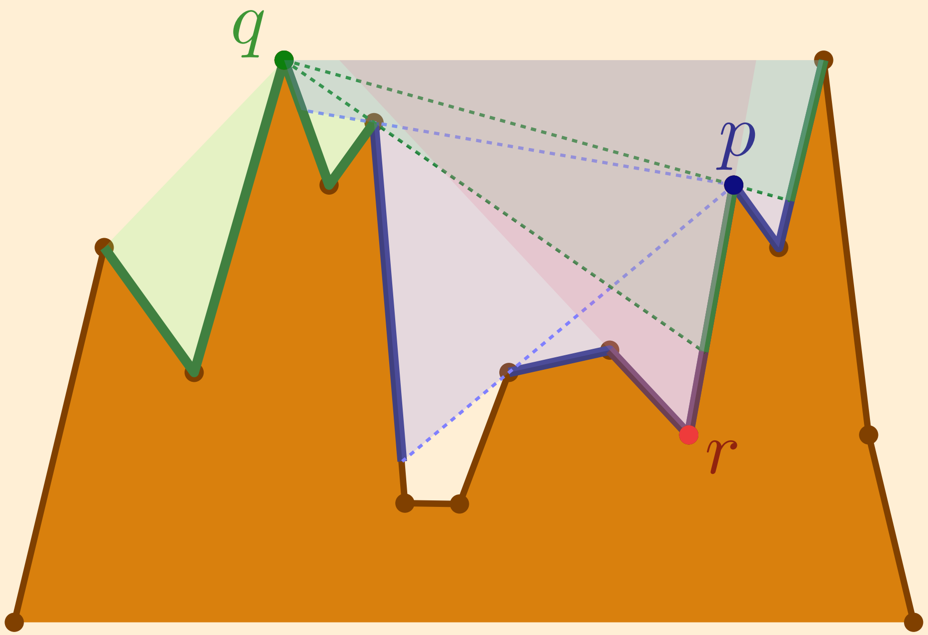


2.5D Terrain $\mathcal{T}$

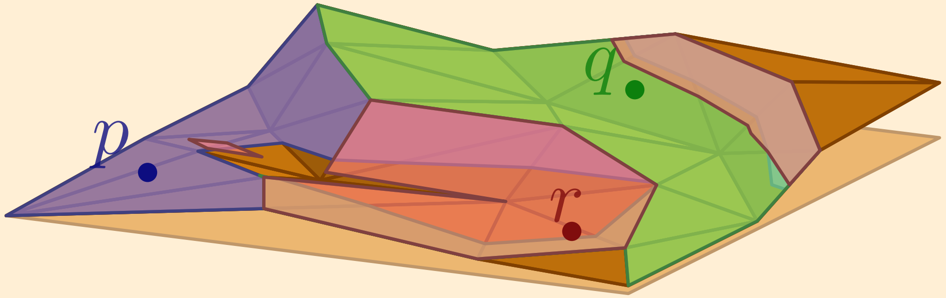
Terrain Visibility with Multiple Viewpoints

Given a set $\mathcal{P}$ of viewpoints

- Is a query point s visible?
- Which viewpoints see s?
- What is the closest viewpoint to s?



1.5D Terrain $\mathcal{T}$



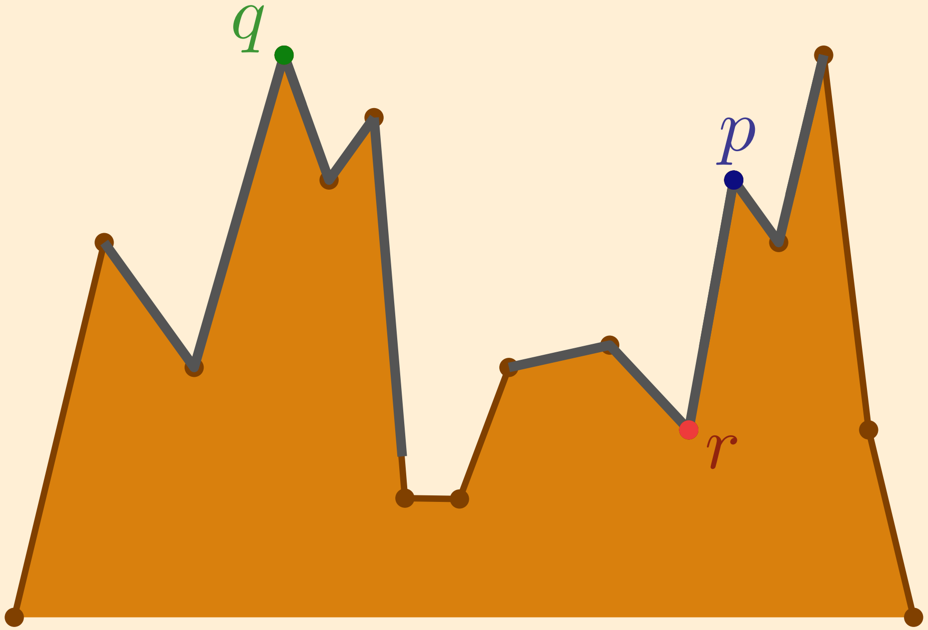
2.5D Terrain $\mathcal{T}$

Terrain Visibility with Multiple Viewpoints

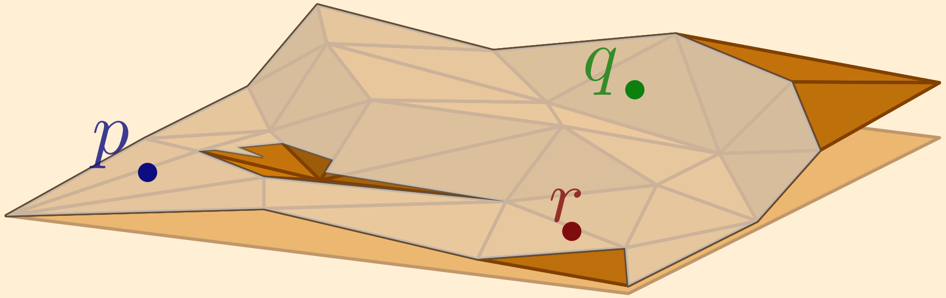
Given a set $\mathcal{P}$ of viewpoints

- Is a query point s visible?
- Which viewpoints see s?
- What is the closest viewpoint to s?

$\text{Vis}(\mathcal{T}, \mathcal{P})$



1.5D Terrain $\mathcal{T}$



2.5D Terrain $\mathcal{T}$

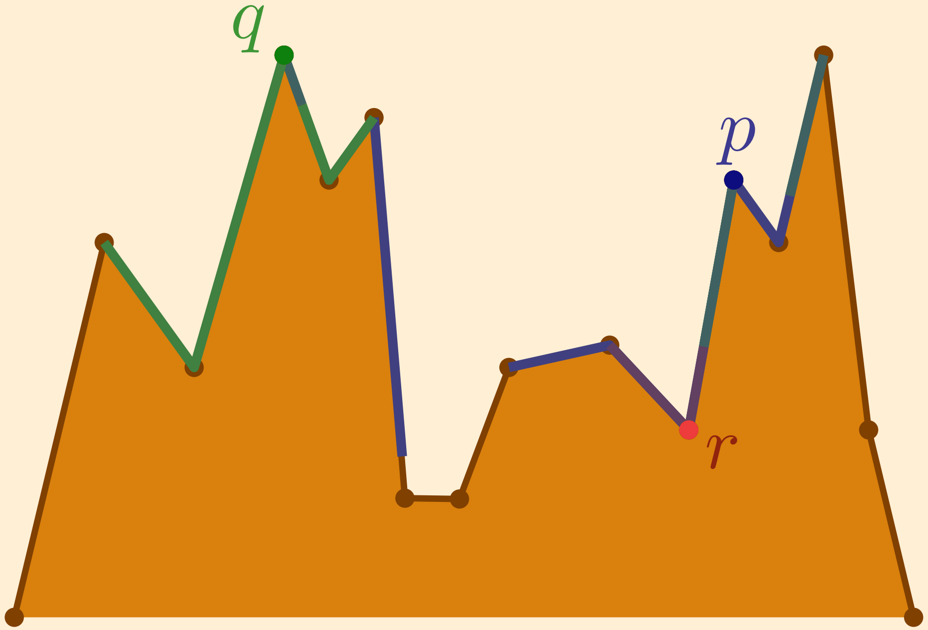
Terrain Visibility with Multiple Viewpoints

Given a set $\mathcal{P}$ of viewpoints

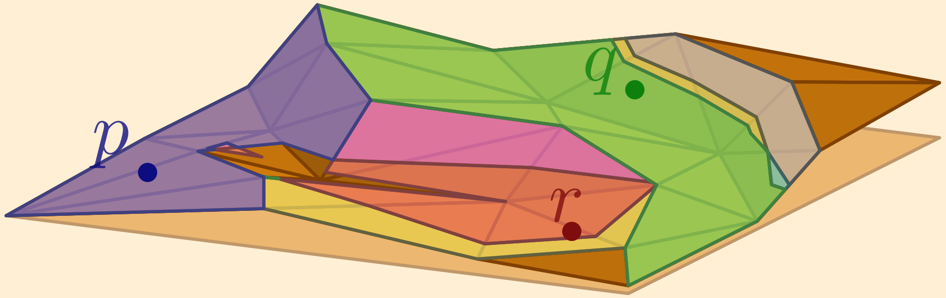
Is a query point s visible? $\mathrm{Vis}(\mathcal{T}, \mathcal{P})$

Which viewpoints see s? $\mathrm{ColVis}(\mathcal{T}, \mathcal{P})$

What is the closest viewpoint to s?



1.5D Terrain $\mathcal{T}$



2.5D Terrain $\mathcal{T}$

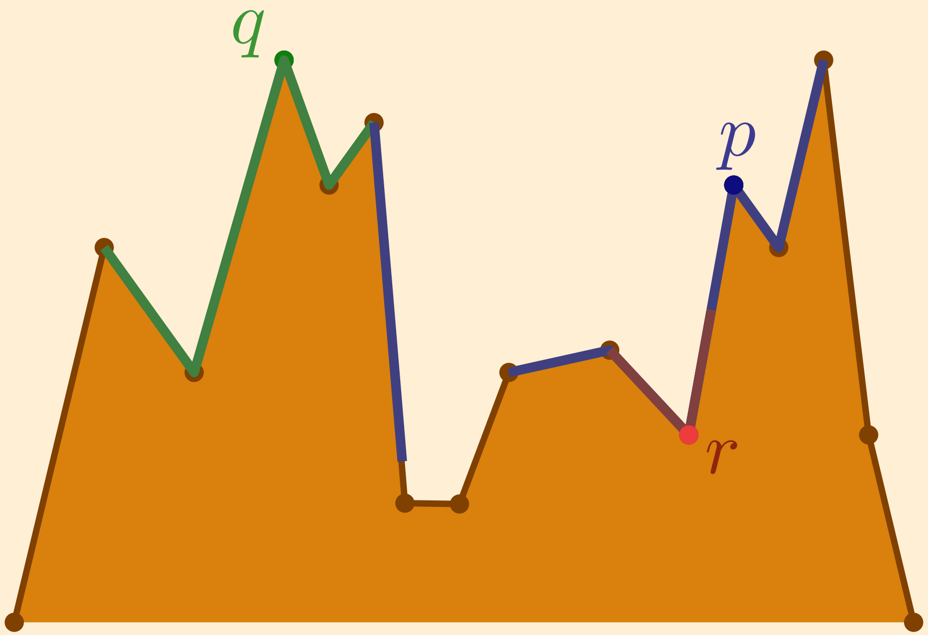
Terrain Visibility with Multiple Viewpoints

Given a set $\mathcal{P}$ of viewpoints

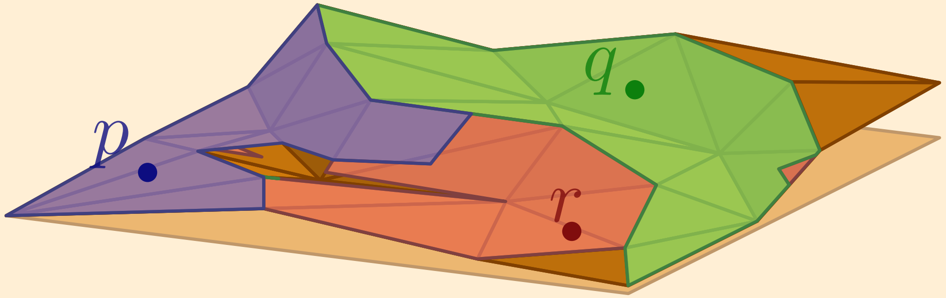
Is a query point s visible? $\mathrm{Vis}(\mathcal{T}, \mathcal{P})$

Which viewpoints see s? $\mathrm{ColVis}(\mathcal{T}, \mathcal{P})$

What is the closest viewpoint to s? $\mathrm{VorVis}(\mathcal{T}, \mathcal{P})$



1.5D Terrain $\mathcal{T}$



2.5D Terrain $\mathcal{T}$

Terrain Visibility with Multiple Viewpoints

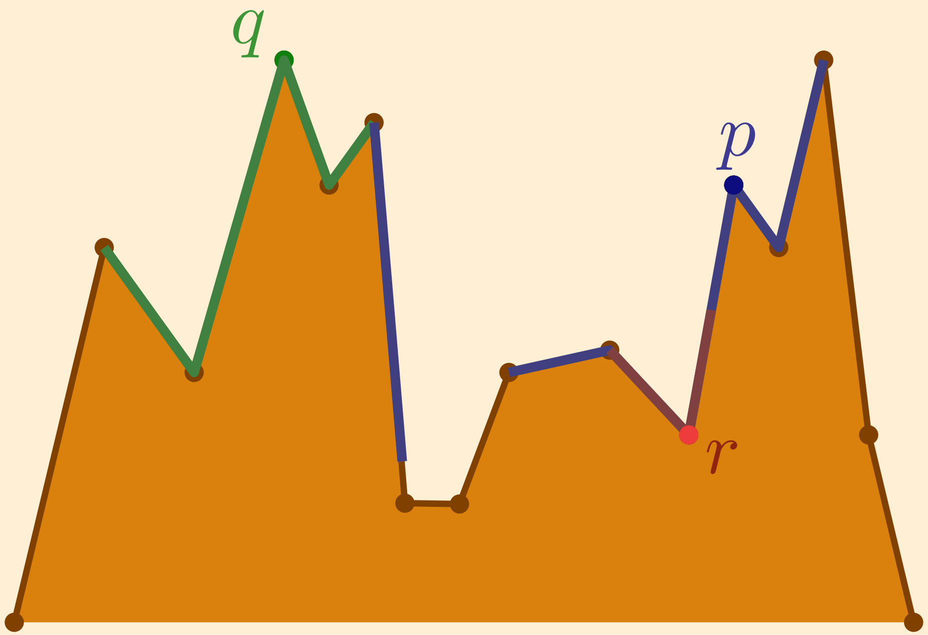
What is the complexity?

How to compute?

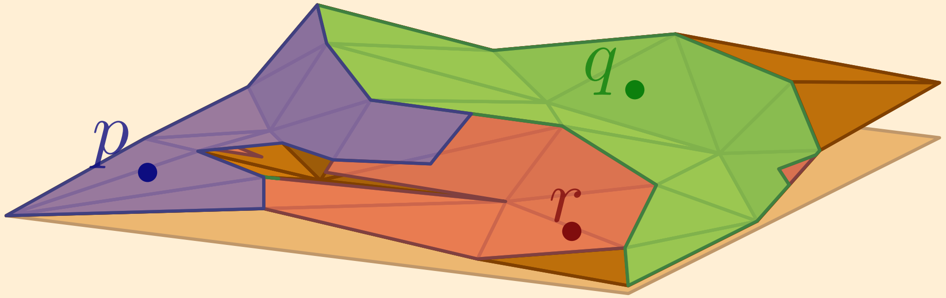
$\mathrm{Vis}(\mathcal{T}, \mathcal{P})$

$\mathrm{ColVis}(\mathcal{T}, \mathcal{P})$

$\mathrm{VorVis}(\mathcal{T}, \mathcal{P})$



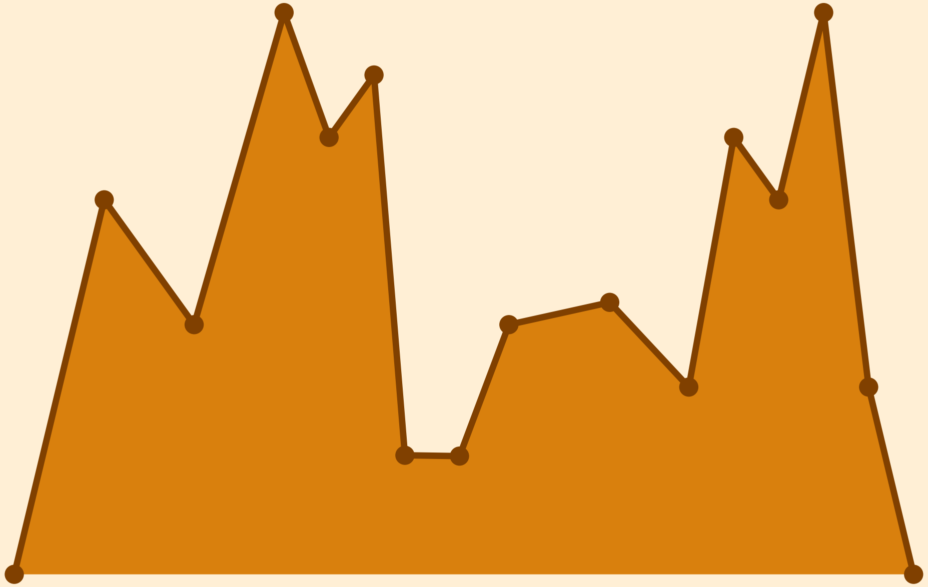
1.5D Terrain $\mathcal{T}$



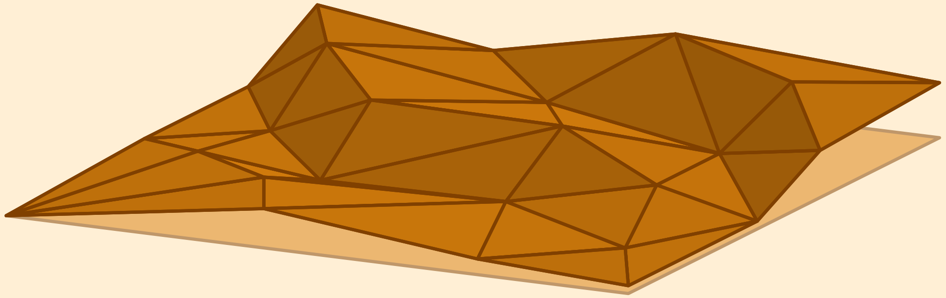
2.5D Terrain $\mathcal{T}$

Related Work

Vis, ColVis, and VorVis for one viewpoint?



1.5D Terrain $\mathcal{T}$



2.5D Terrain $\mathcal{T}$

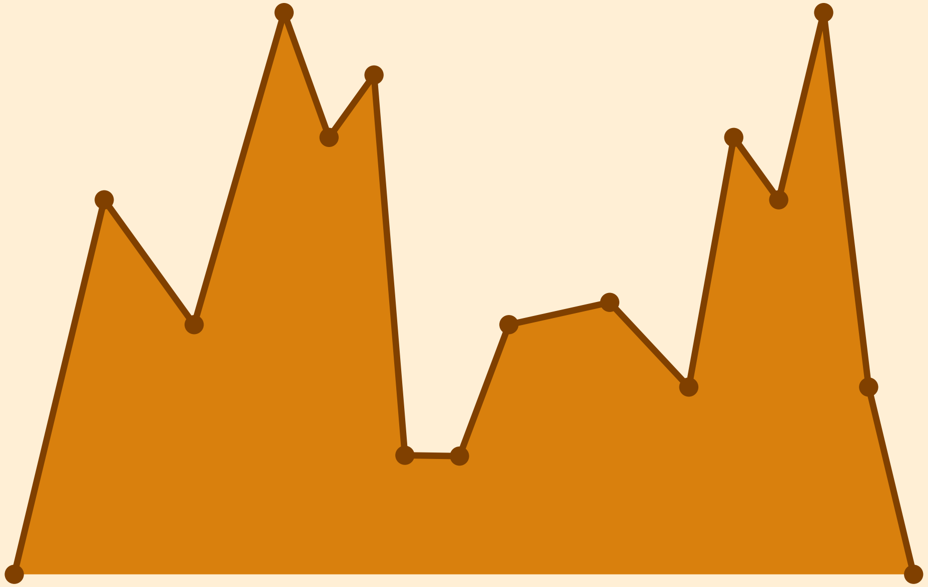
Related Work

Vis, ColVis, and VorVis for one viewpoint?

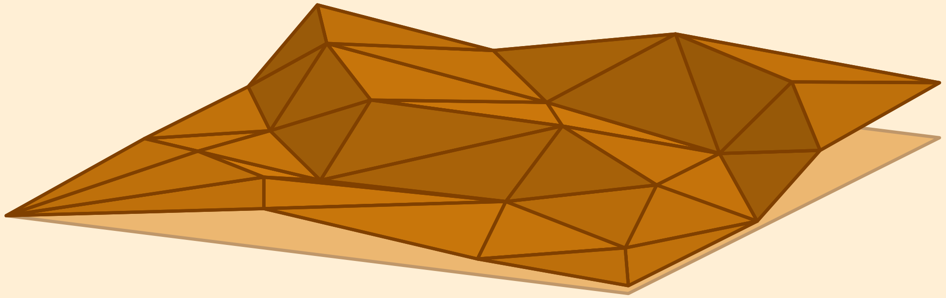
$\mathrm{Vis}(\mathcal{T}, \{p\}) = \mathcal{V}_{\mathcal{T}}(p)$

Complexity:	$\Theta(n)$	$\Theta(n^2)$
Algorithm:	$O(n)$	$O((n\alpha(n) + k) \log n)$

[Katz *et al.*, Comp. Geom '92]



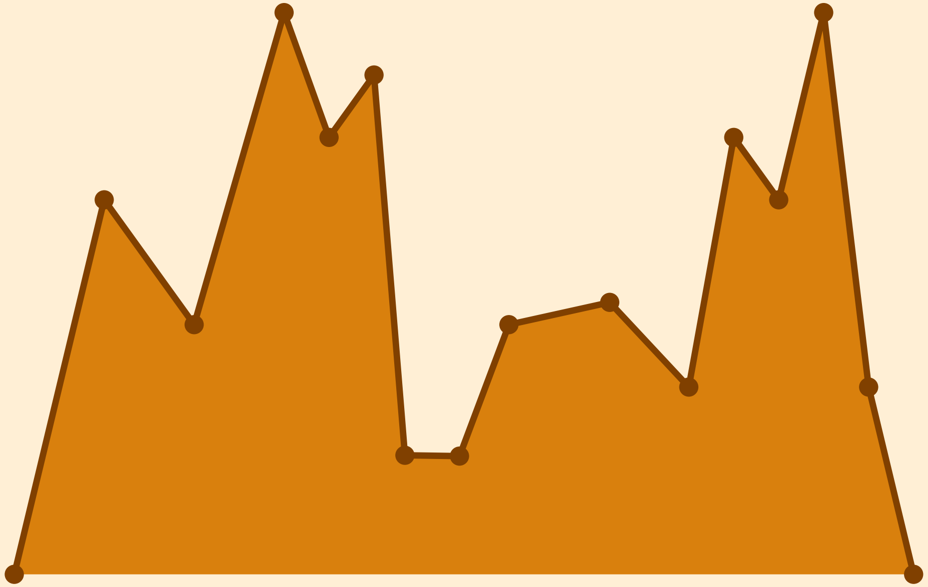
1.5D Terrain $\mathcal{T}$



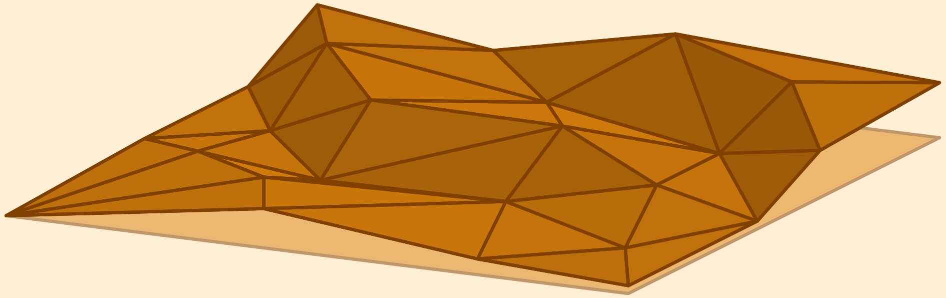
2.5D Terrain $\mathcal{T}$

Related Work

Where to place viewpoints/guards/observers?



1.5D Terrain $\mathcal{T}$



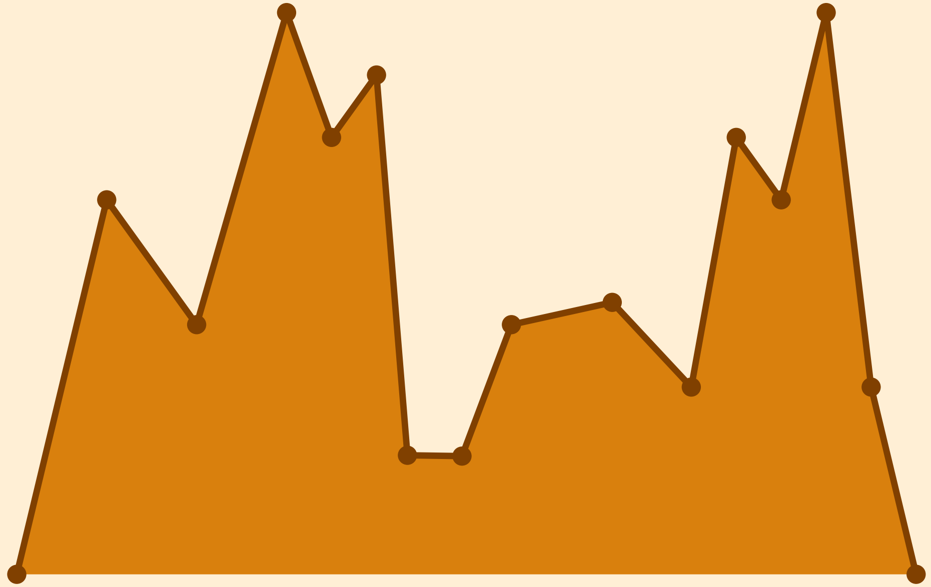
2.5D Terrain $\mathcal{T}$

Related Work

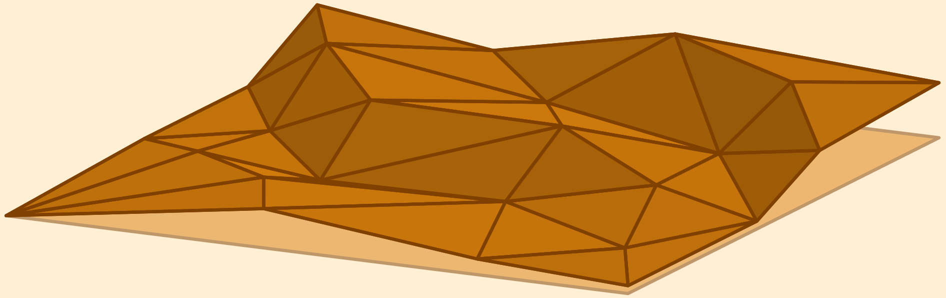
Where to place viewpoints/guards/observers?

NP-Hard

[King and Krohn, SODA'10]



1.5D Terrain $\mathcal{T}$



2.5D Terrain $\mathcal{T}$

Related Work

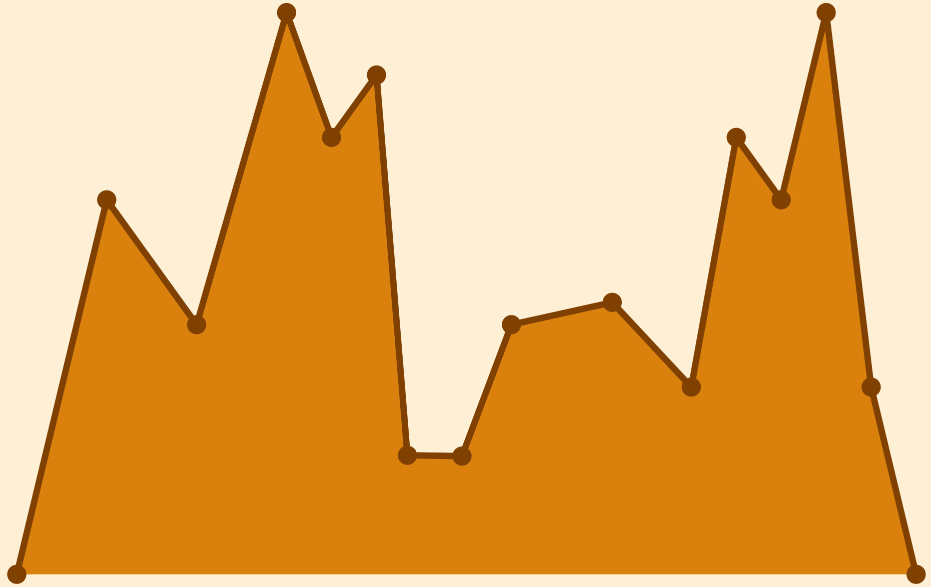
Where to place viewpoints/guards/observers?

NP-Hard

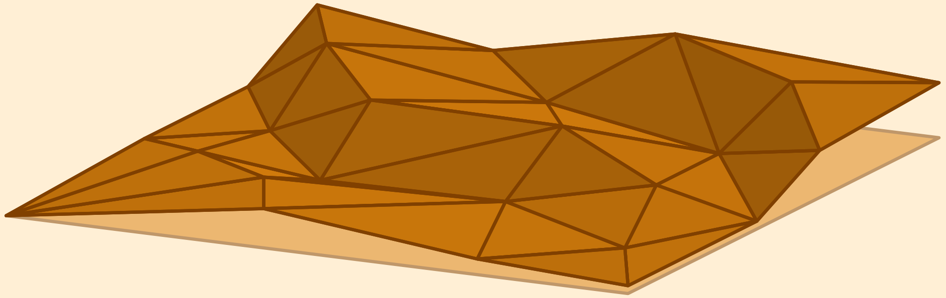
[King and Krohn, SODA'10]

$(1+\varepsilon)$-approx.

[Gibson *et al.*, APPROX-RANDOM'09].



1.5D Terrain $\mathcal{T}$



2.5D Terrain $\mathcal{T}$

Related Work

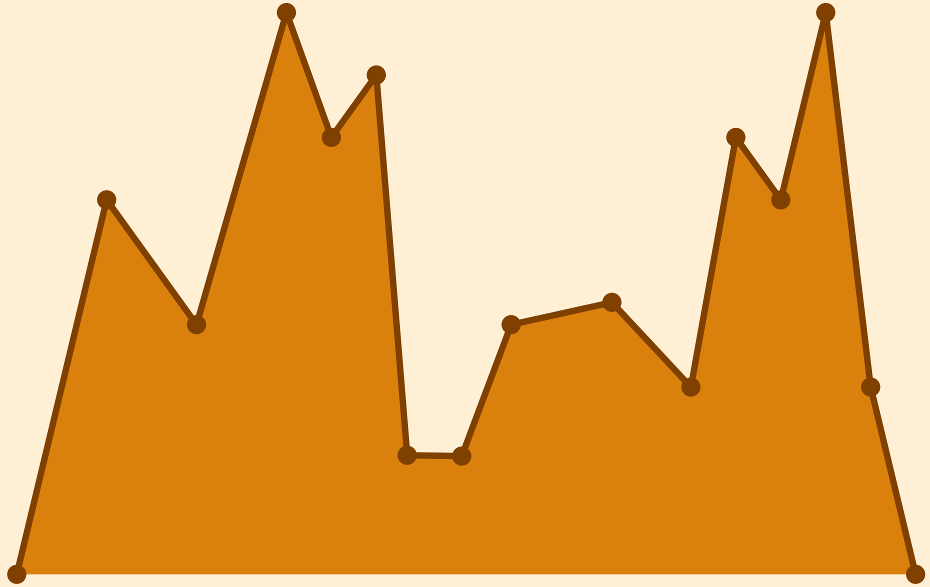
Where to place viewpoints/guards/observers?

NP-Hard
[King and Krohn, SODA'10]

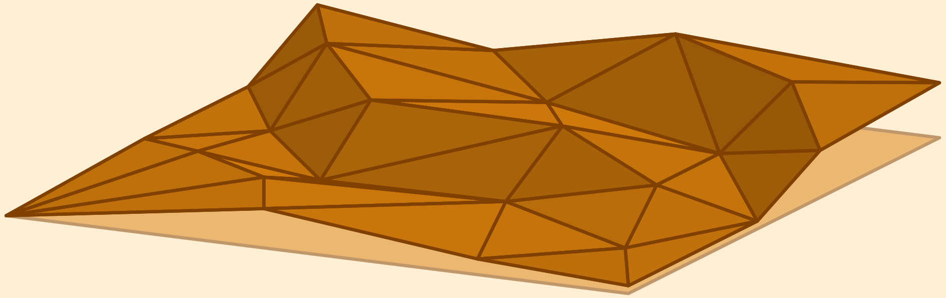
$(1+\varepsilon)$-approx.
[Gibson *et al.*, APPROX-RANDOM'09].

NP-Hard
[Cole & Sharir, J. Sym. Comp '89]

Hard to approximate
[Eidenbenz *et al.*, Algoritmica '00]



1.5D Terrain $\mathcal{T}$

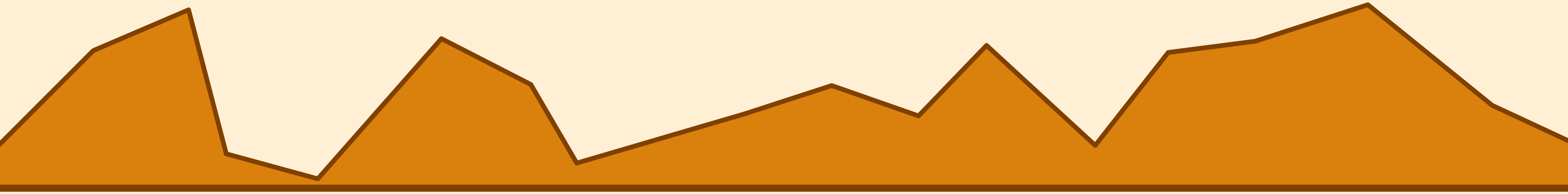


2.5D Terrain $\mathcal{T}$

Results

n = size $\mathcal{T}$
m = size $\mathcal{P}$

Structure	Max. size	Computation time
Vis	$\Theta(n)$	
ColVis	$\Theta(mn)$	
VorVis	$\Theta(mn)$	

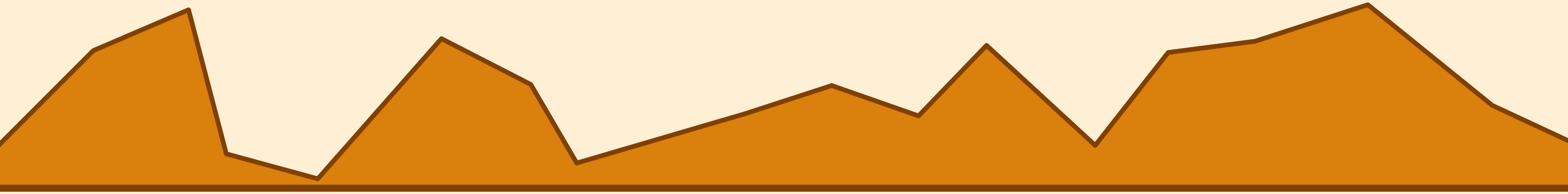


Results

n = size $\mathcal{T}$
m = size $\mathcal{P}$

k_c = complex. ColVis
k_v = complex. VorVis

Structure	Max. size	Computation time
Vis	$\Theta(n)$	$O(n \log n)$
ColVis	$\Theta(mn)$	$O(n + (m^2 + k_c) \log n)$
VorVis	$\Theta(mn)$	$O(n + (m^2 + k_c) \log n + k_v(m + \log n \log m))$

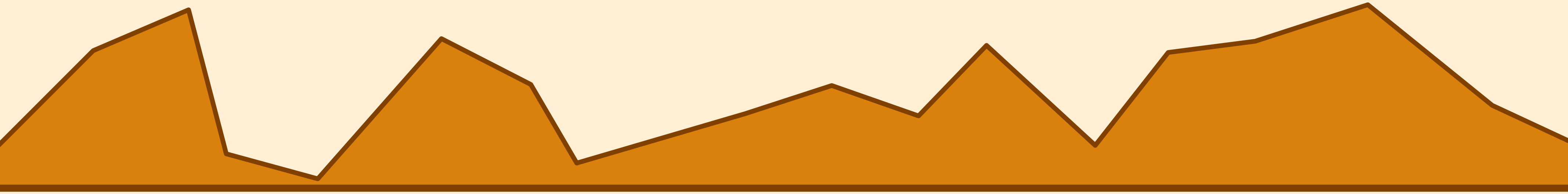


Results

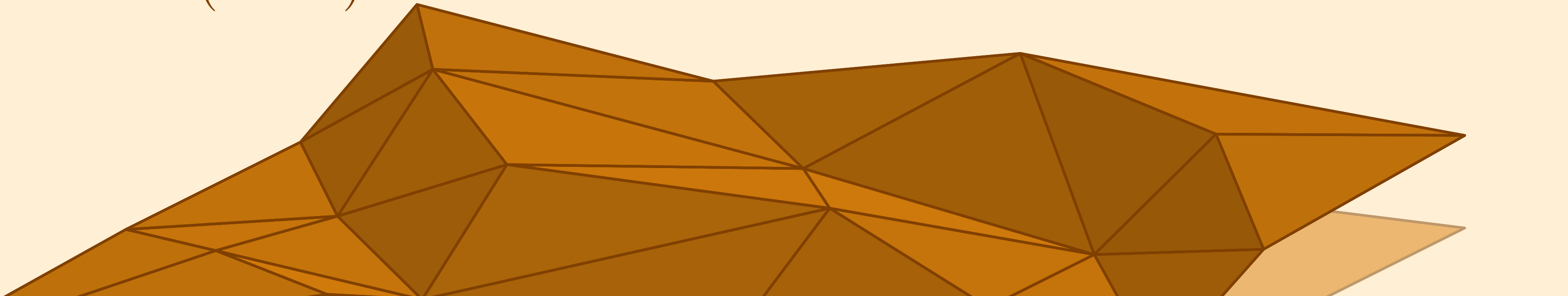
n = size $\mathcal{T}$
m = size $\mathcal{P}$

k_c = complex. ColVis
k_v = complex. VorVis

Structure	Max. size	Computation time
Vis	$\Theta(n)$	$O(n \log n)$
ColVis	$\Theta(mn)$	$O(n + (m^2 + k_c) \log n)$
VorVis	$\Theta(mn)$	$O(n + (m^2 + k_c) \log n + k_v(m + \log n \log m))$



Structure	Max. size	Computation time
Vis	$O(m^3n^2)$	Lower bound: $\Omega(m^2n^2)$
ColVis	$O(m^3n^2)$	
VorVis	$O(m^4n^2)$	

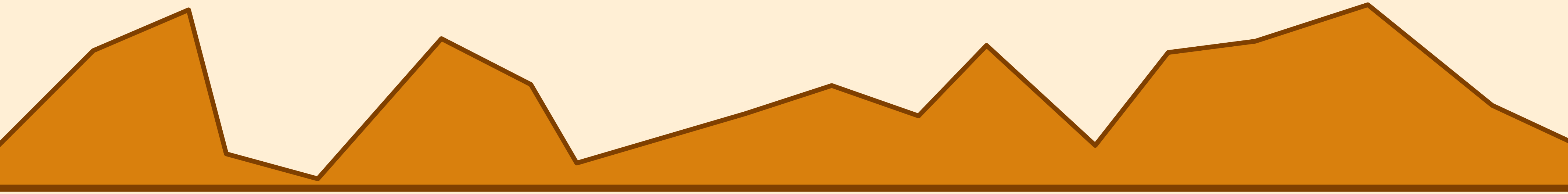


Results

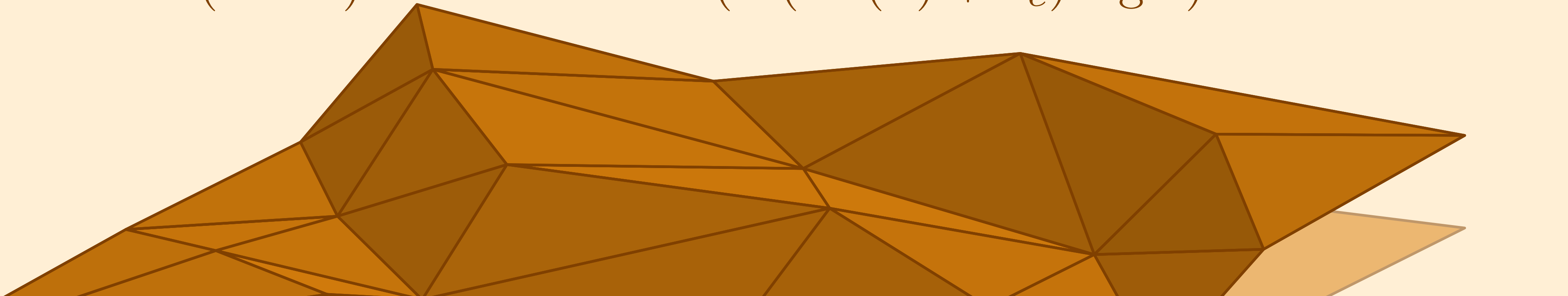
n = size $\mathcal{T}$
m = size $\mathcal{P}$

k_c = complex. ColVis
k_v = complex. VorVis

Structure	Max. size	Computation time
Vis	$\Theta(n)$	$O(n \log n)$
ColVis	$\Theta(mn)$	$O(n + (m^2 + k_c) \log n)$
VorVis	$\Theta(mn)$	$O(n + (m^2 + k_c) \log n + k_v(m + \log n \log m))$



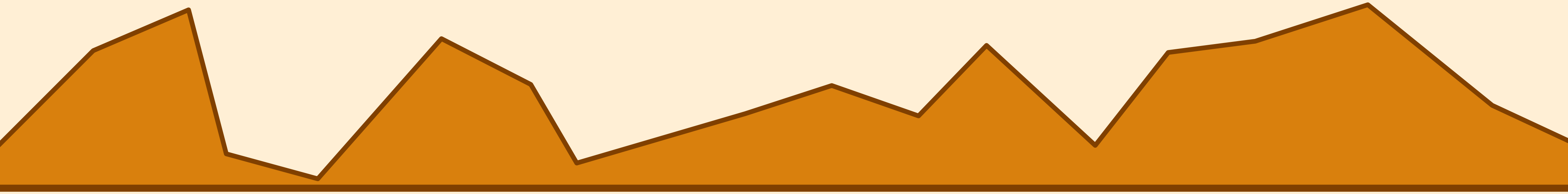
Structure	Max. size	Computation time
Vis	$O(m^3n^2)$	$O(m(n\alpha(n) + k_c) \log n)$
ColVis	$O(m^3n^2)$	$O(m(n\alpha(n) + k_c) \log n)$
VorVis	$O(m^4n^2)$	$O(m(n\alpha(n) + k_c) \log n)$



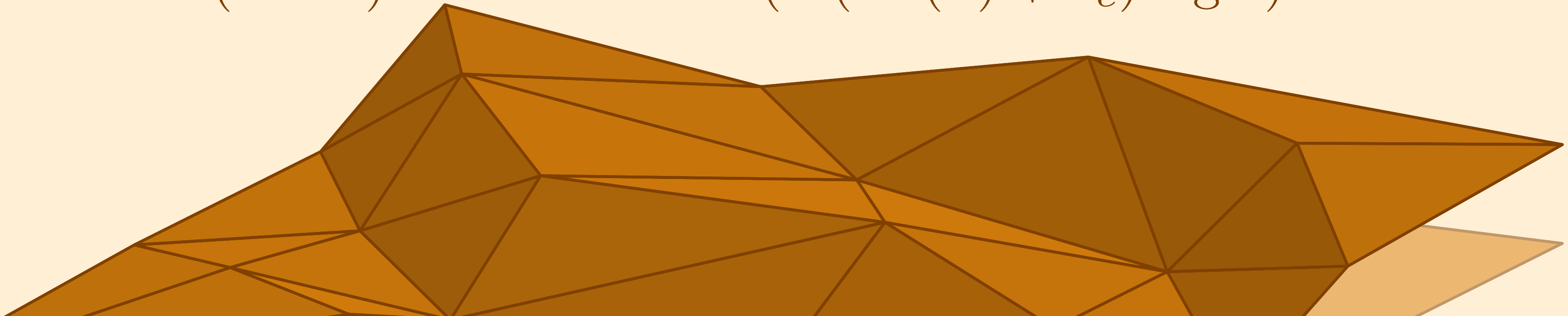
Results

n = size $\mathcal{T}$ k_c = complex. ColVis
m = size $\mathcal{P}$ k_v = complex. VorVis

Structure	Max. size	Computation time
Vis	$\Theta(n)$	$O(n \log n)$
ColVis	$\Theta(mn)$	$O(n + (m^2 + k_c) \log n)$
VorVis	$\Theta(mn)$	$O(n + (m^2 + k_c) \log n + k_v(m + \log n \log m))$



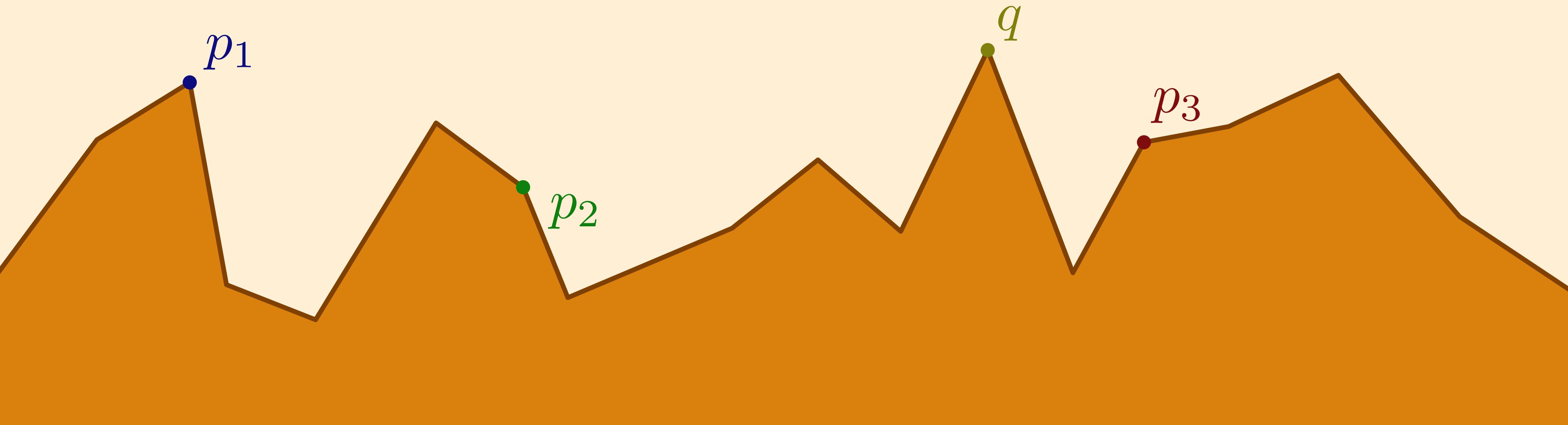
Structure	Max. size	Computation time
Vis	$O(m^3 n^2)$	$O(m(n\alpha(n) + k_c) \log n)$
ColVis	$O(m^3 n^2)$	$O(m(n\alpha(n) + k_c) \log n)$
VorVis	$O(m^4 n^2)$	$O(m(n\alpha(n) + k_c) \log n)$



Computing Vis

We compute *left* Vis:

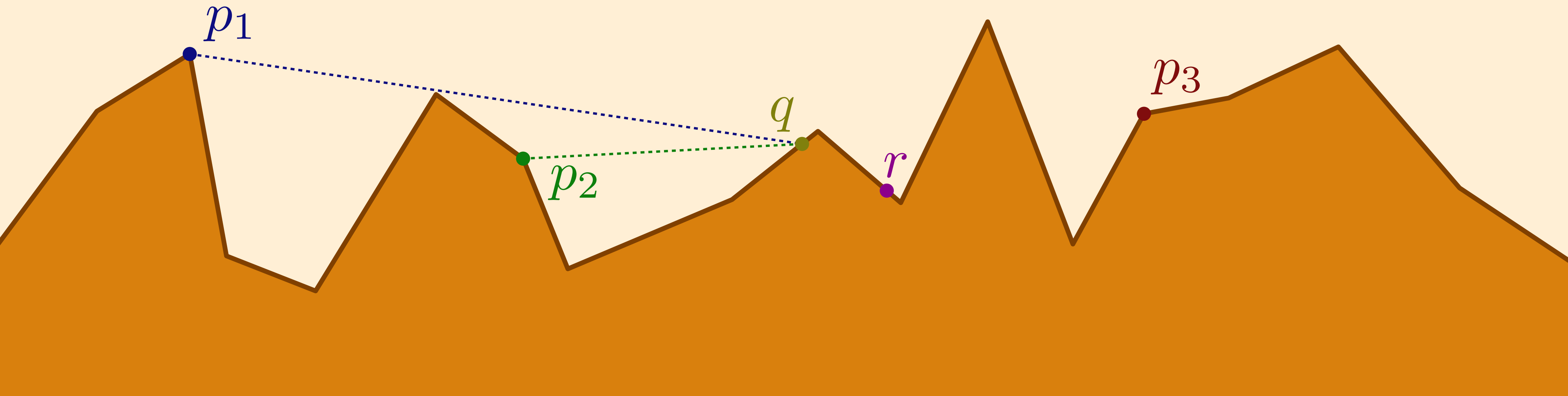
$q \in \textit{left}\,\mathrm{Vis} \iff q$ visible by a viewpoint to the left of q.



Computing Vis

We compute $\mathit{left}\,\mathrm{Vis}$:
$q \in \mathit{left}\,\mathrm{Vis} \iff q$ visible by a viewpoint to the left of q.

Observation 1. *Let q be left-visible from p_1 and p_2. For any r to the right of q:*

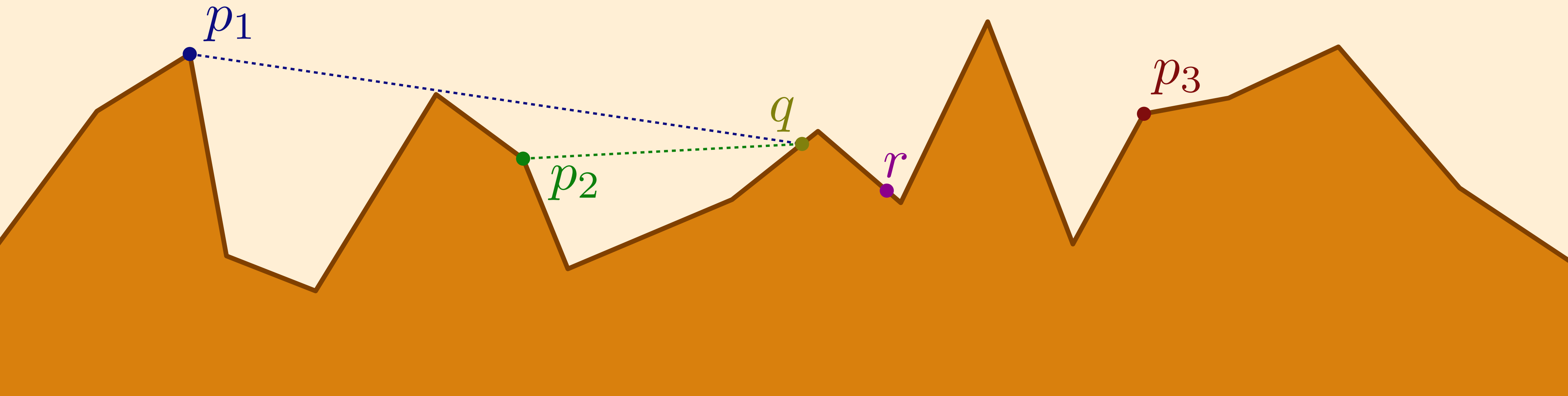


Computing Vis

We compute $\mathit{left}\,\mathrm{Vis}$:

$q \in \mathit{left}\,\mathrm{Vis} \iff q$ visible by a viewpoint to the left of q.

Observation 1. *Let q be left-visible from p_1 and p_2. For any r to the right of q: p_1 does not see $r \implies p_2$ does not see r.*

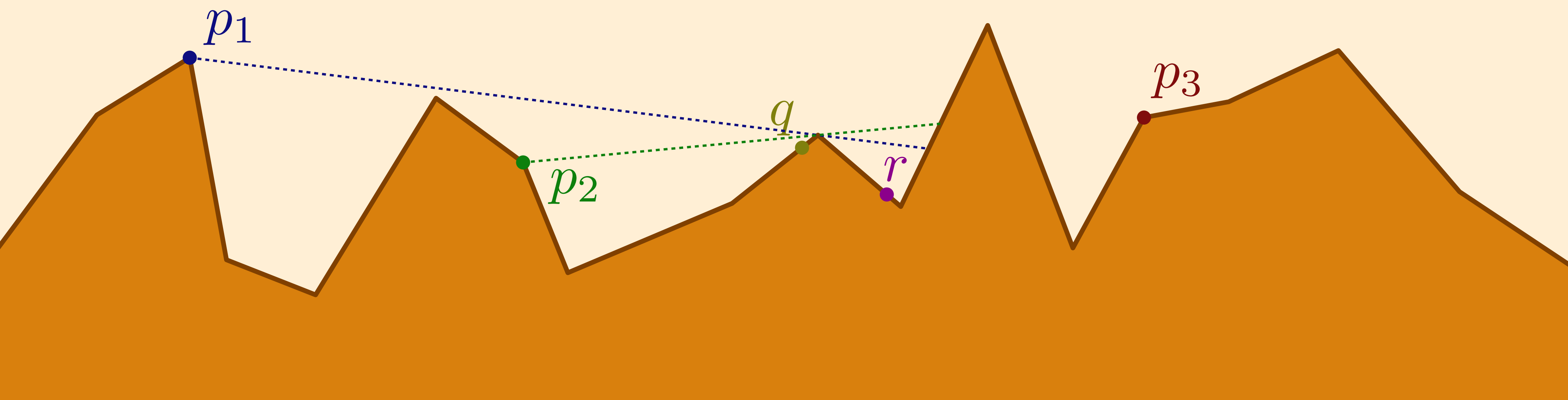


Computing Vis

We compute *left* Vis:

$q \in \mathit{left}\,\mathrm{Vis} \iff q$ visible by a viewpoint to the left of q.

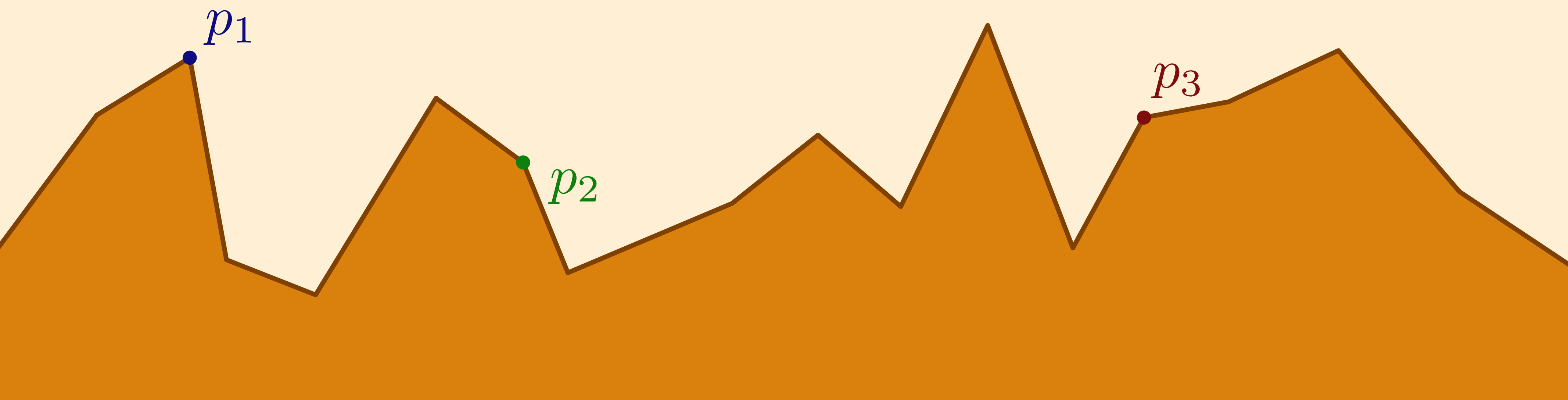
Observation 1. *Let q be left-visible from p_1 and p_2. For any r to the right of q: p_1 does not see $r \implies p_2$ does not see r.*



Computing Vis

We compute *left* Vis:
$q \in \textit{left}\,\mathrm{Vis} \iff q$ visible by a viewpoint to the left of q.

Algorithm: Sweep the terrain, maintain leftmost visible viewpoint p_a.

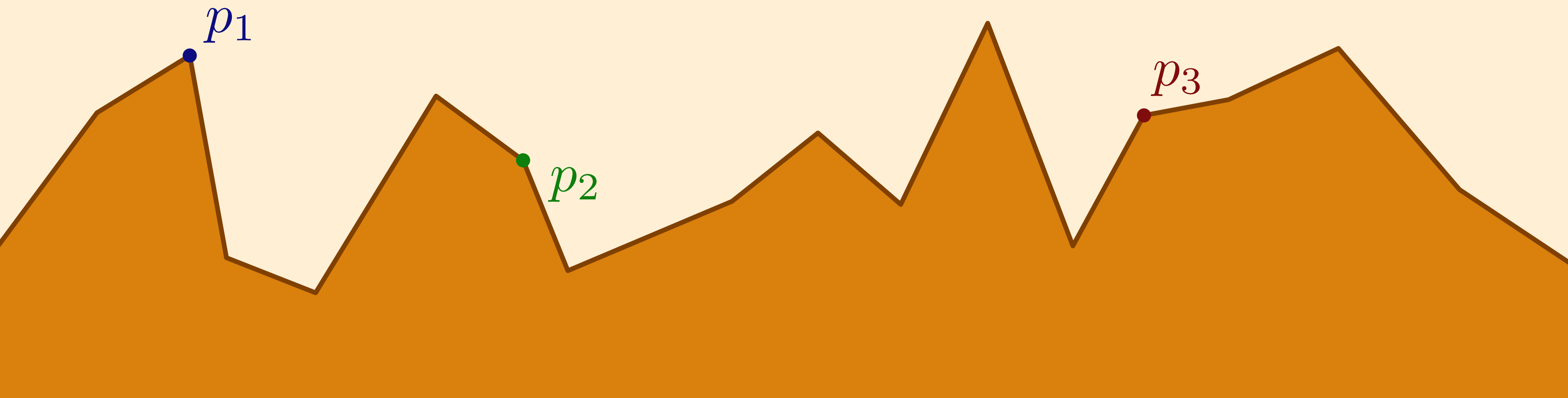


Computing Vis

We compute *left* Vis:
$q \in \textit{left}\,\mathrm{Vis} \iff q$ visible by a viewpoint to the left of q.

Algorithm: Sweep the terrain, maintain leftmost visible viewpoint p_a.

- Viewpoint events
- Vertex events



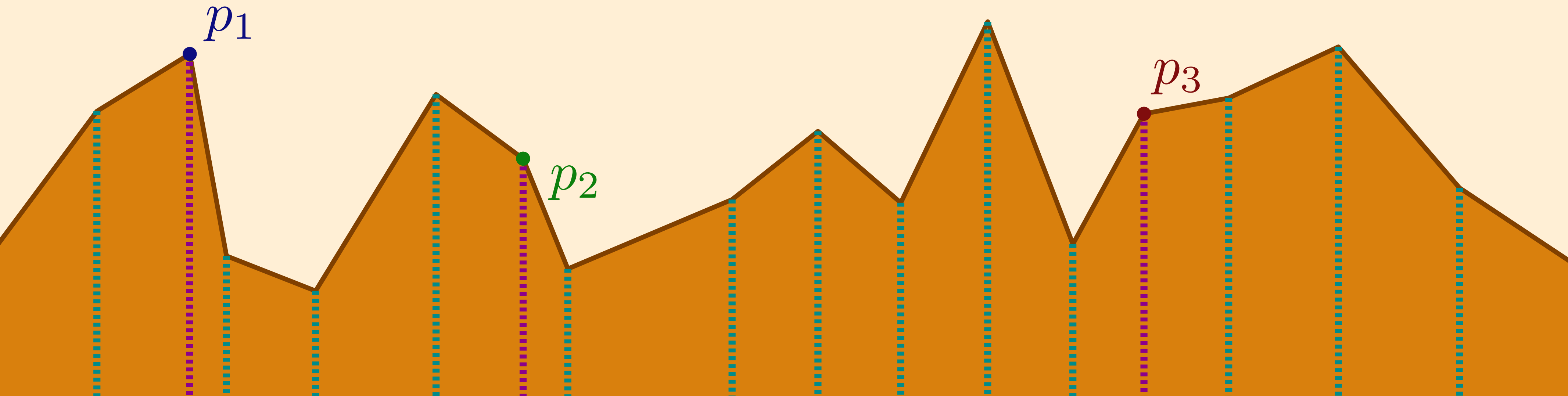
Computing Vis

We compute *left* Vis:
$q \in \textit{left}\,\mathrm{Vis} \iff q$ visible by a viewpoint to the left of q.

Algorithm: Sweep the terrain, maintain leftmost visible viewpoint p_a.

- Viewpoint events
- Vertex events

p_a: $\perp$

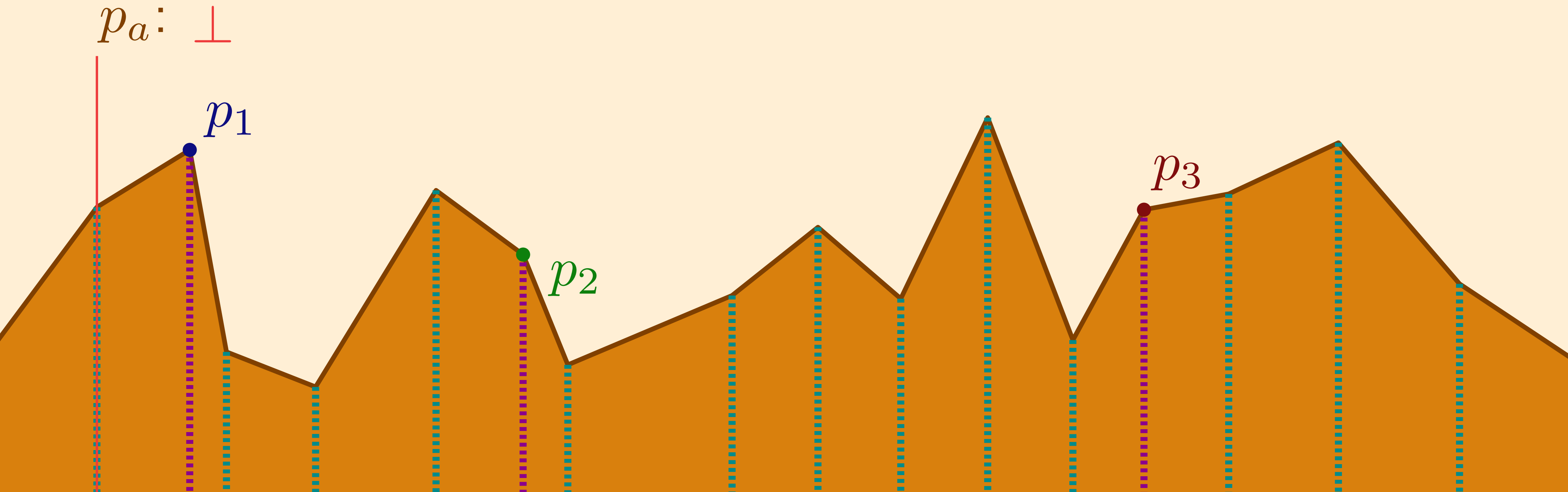


Computing Vis

We compute *left* Vis:
$q \in \textit{left}\,\mathrm{Vis} \iff q$ visible by a viewpoint to the left of q.

Algorithm: Sweep the terrain, maintain leftmost visible viewpoint p_a.

- Viewpoint events
- Vertex events



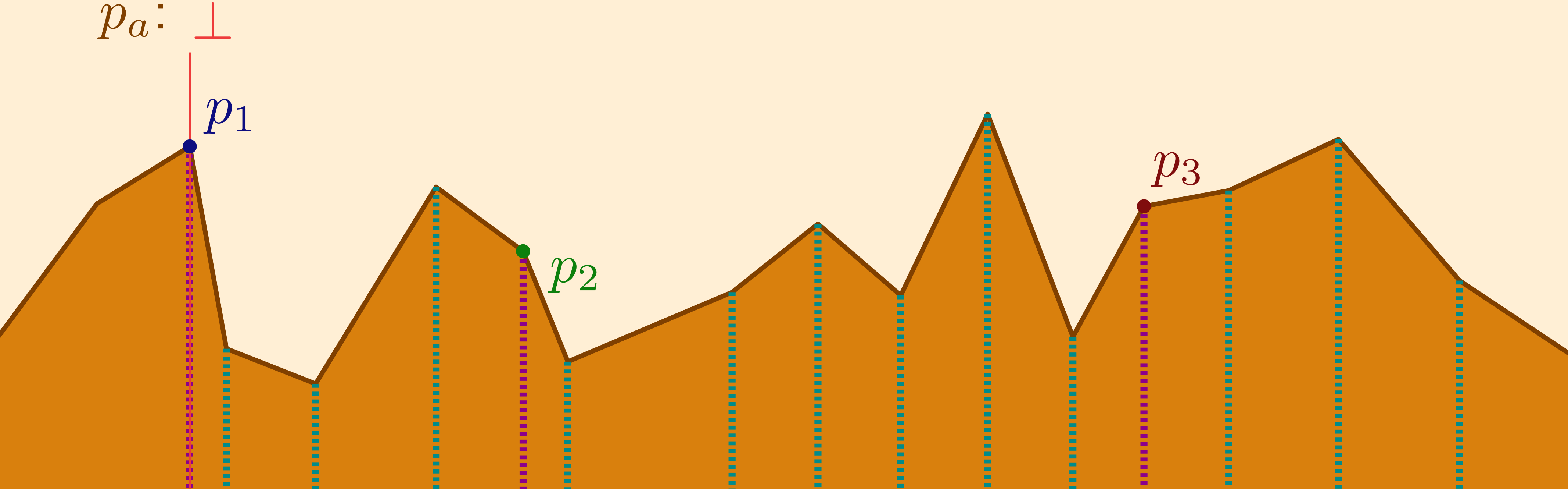
Computing Vis

We compute *left* Vis:

$q \in \textit{left}\,\mathrm{Vis} \iff q$ visible by a viewpoint to the left of q.

Algorithm: Sweep the terrain, maintain leftmost visible viewpoint p_a.

- Viewpoint events
- Vertex events

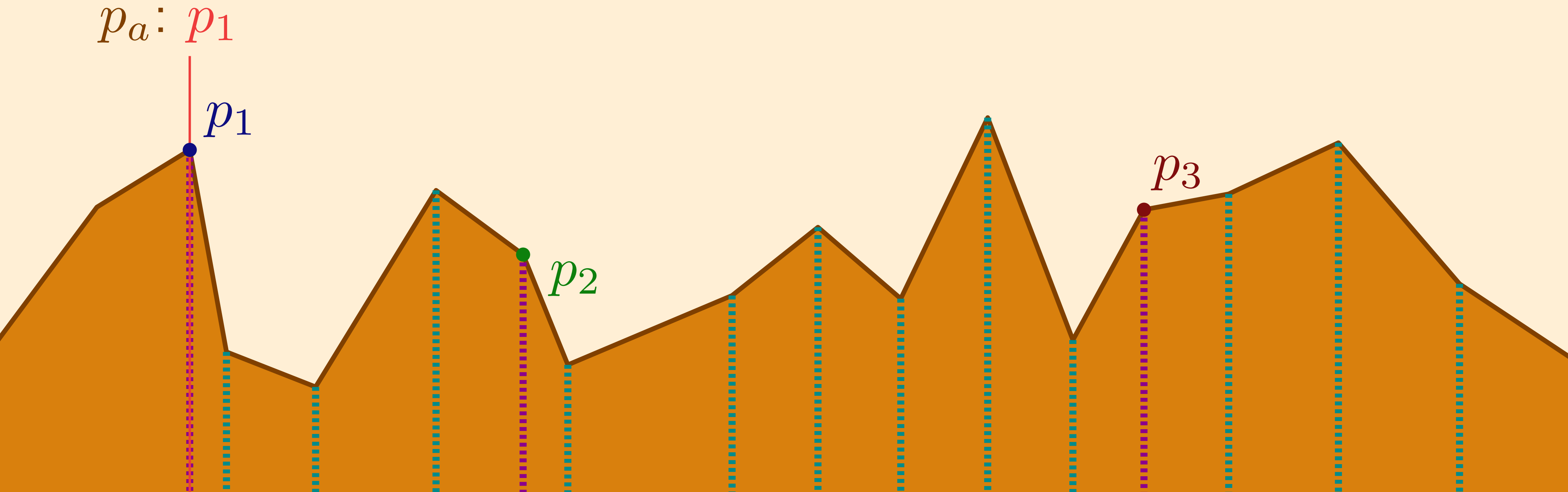


Computing Vis

We compute *left* Vis:
$q \in \textit{left}\,\mathrm{Vis} \iff q$ visible by a viewpoint to the left of q.

Algorithm: Sweep the terrain, maintain leftmost visible viewpoint p_a.

- Viewpoint events
- Vertex events



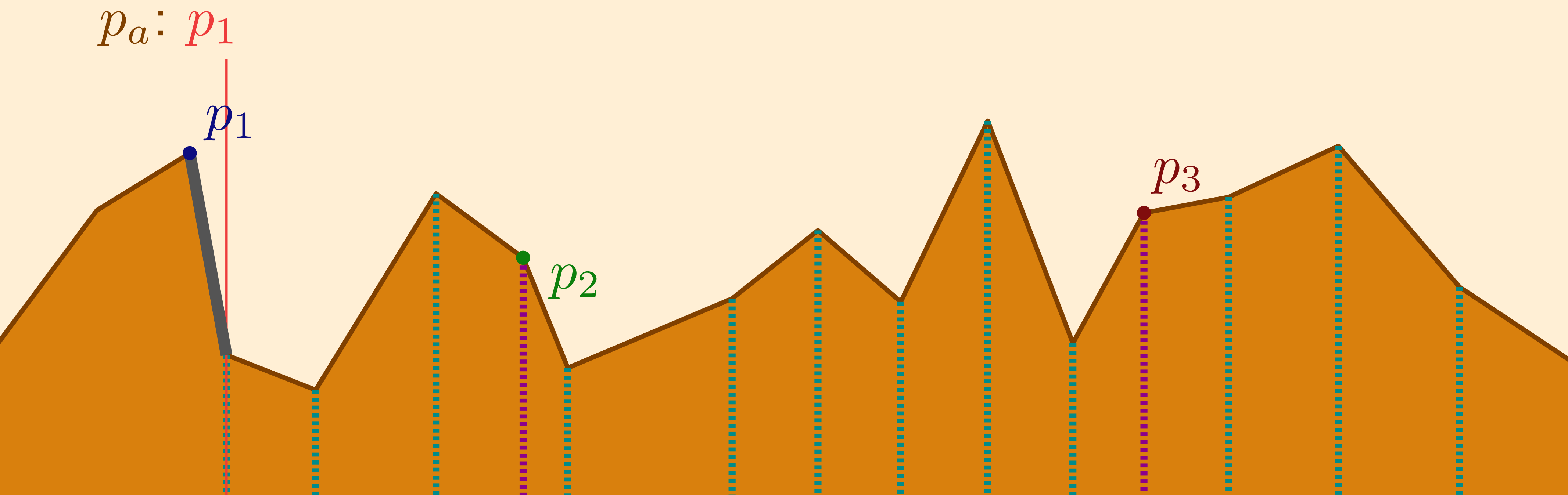
Computing Vis

We compute *left* Vis:

$q \in \mathit{left}\,\mathrm{Vis} \iff q$ visible by a viewpoint to the left of q.

Algorithm: Sweep the terrain, maintain leftmost visible viewpoint p_a.

- Viewpoint events
- Vertex events



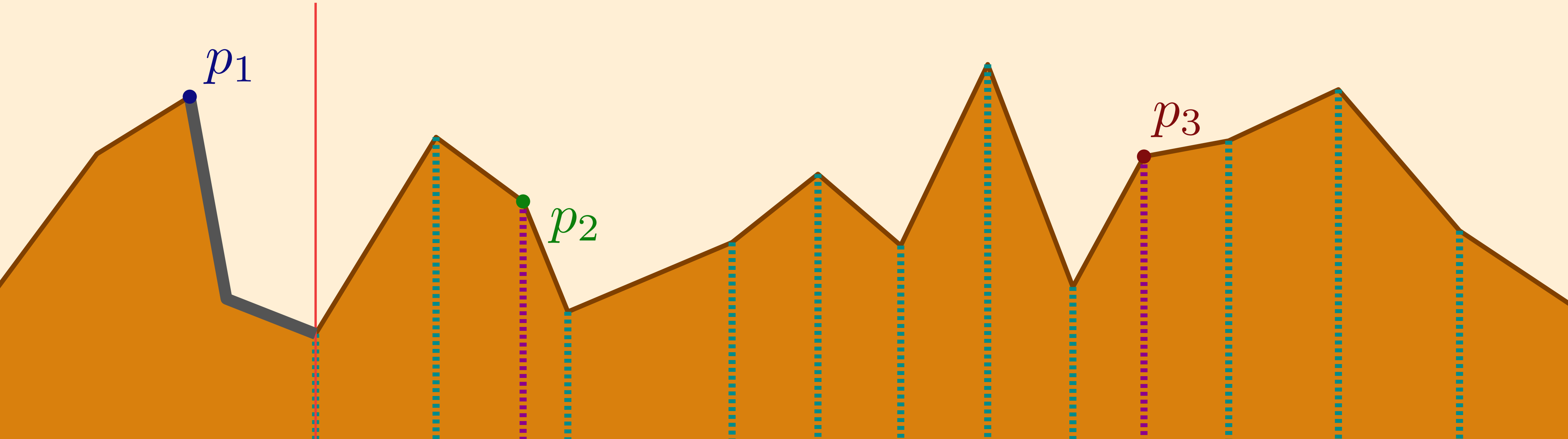
Computing Vis

We compute *left* Vis:
$q \in \mathit{left}\,\mathrm{Vis} \iff q$ visible by a viewpoint to the left of q.

Algorithm: Sweep the terrain, maintain leftmost visible viewpoint p_a.

- Viewpoint events
- Vertex events

p_a: p_1



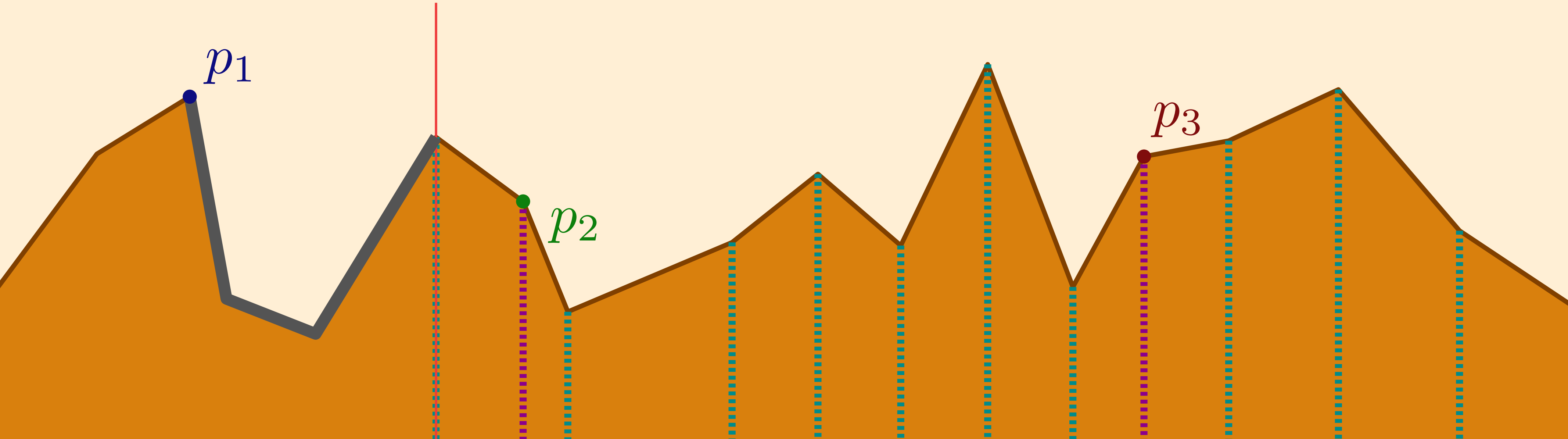
Computing Vis

We compute *left* Vis:
$q \in \textit{left}\,\mathrm{Vis} \iff q$ visible by a viewpoint to the left of q.

Algorithm: Sweep the terrain, maintain leftmost visible viewpoint p_a.

- Viewpoint events
- Vertex events

p_a: p_1

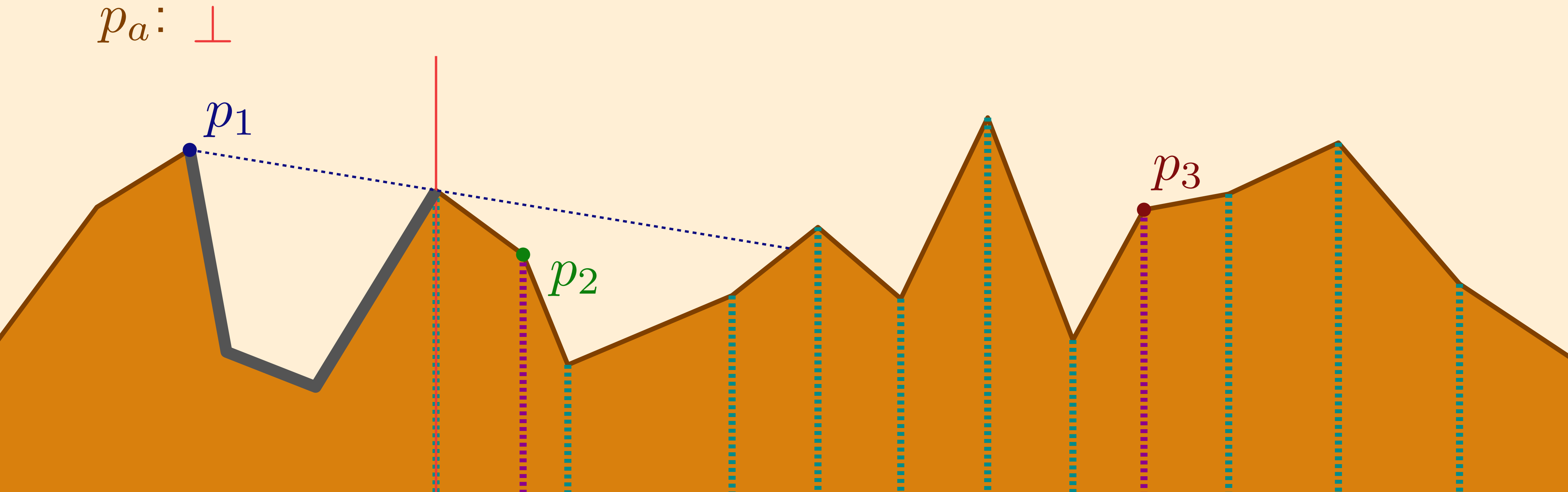


Computing Vis

We compute *left* Vis:
$q \in \textit{left}\,\mathrm{Vis} \iff q$ visible by a viewpoint to the left of q.

Algorithm: Sweep the terrain, maintain leftmost visible viewpoint p_a.

- Viewpoint events
- Vertex events



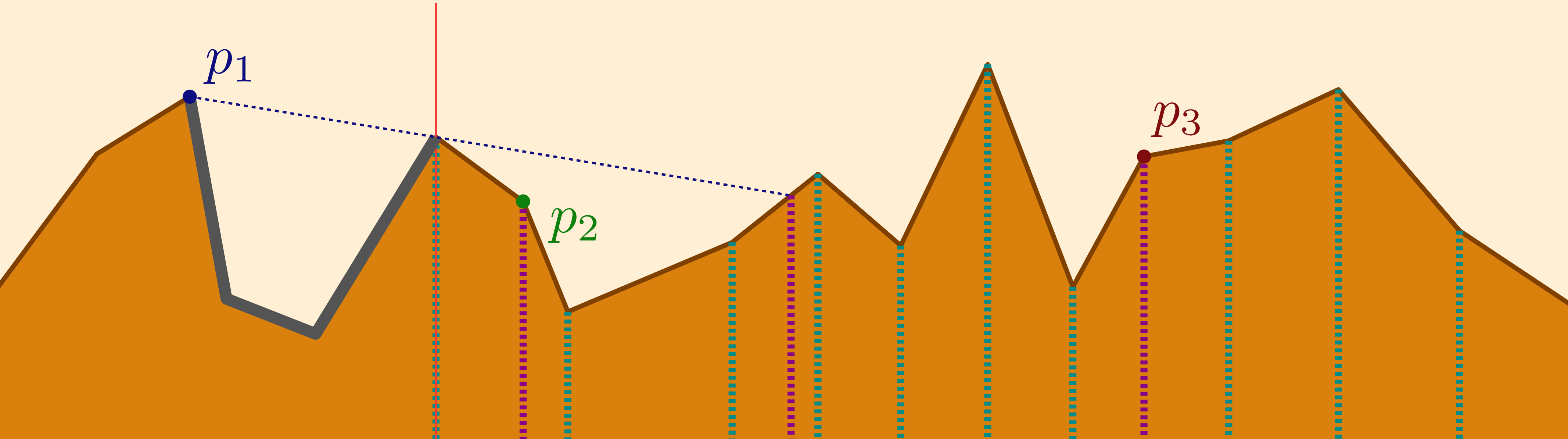
Computing Vis

We compute *left* Vis:
$q \in \mathit{left}\,\mathrm{Vis} \iff q$ visible by a viewpoint to the left of q.

Algorithm: Sweep the terrain, maintain leftmost visible viewpoint p_a.

- Viewpoint events
- Vertex events

p_a: $\perp$



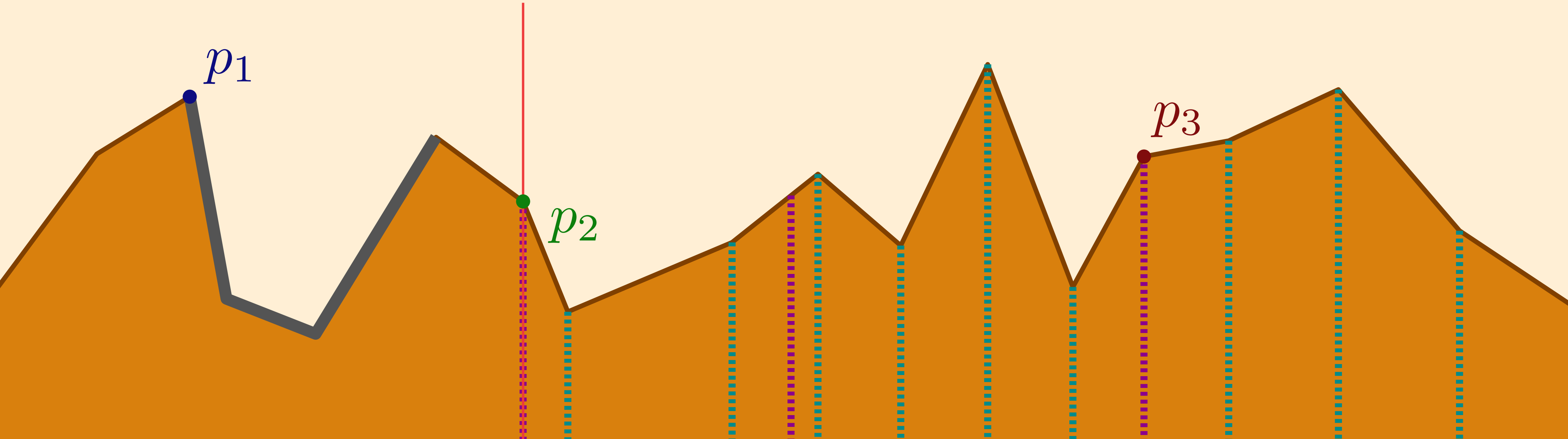
Computing Vis

We compute *left* Vis:
$q \in \textit{left}\,\mathrm{Vis} \iff q$ visible by a viewpoint to the left of q.

Algorithm: Sweep the terrain, maintain leftmost visible viewpoint p_a.

- Viewpoint events
- Vertex events

p_a: p_2



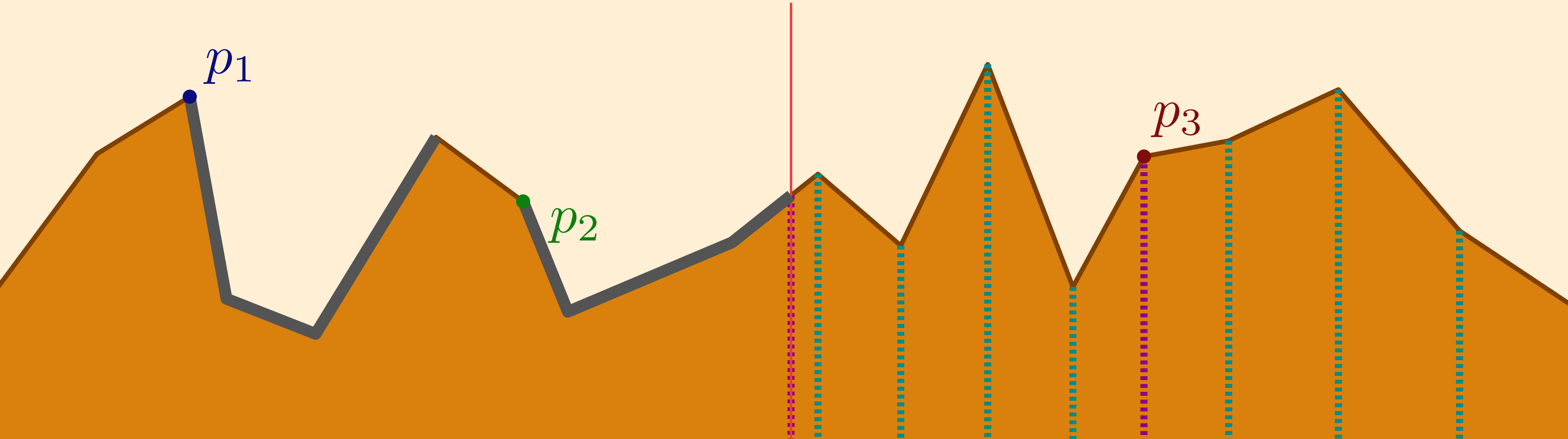
Computing Vis

We compute *left* Vis:
$q \in \textit{left}\ \mathrm{Vis} \iff q$ visible by a viewpoint to the left of q.

Algorithm: Sweep the terrain, maintain leftmost visible viewpoint p_a.

- Viewpoint events
- Vertex events

p_a: p_2



Computing Vis

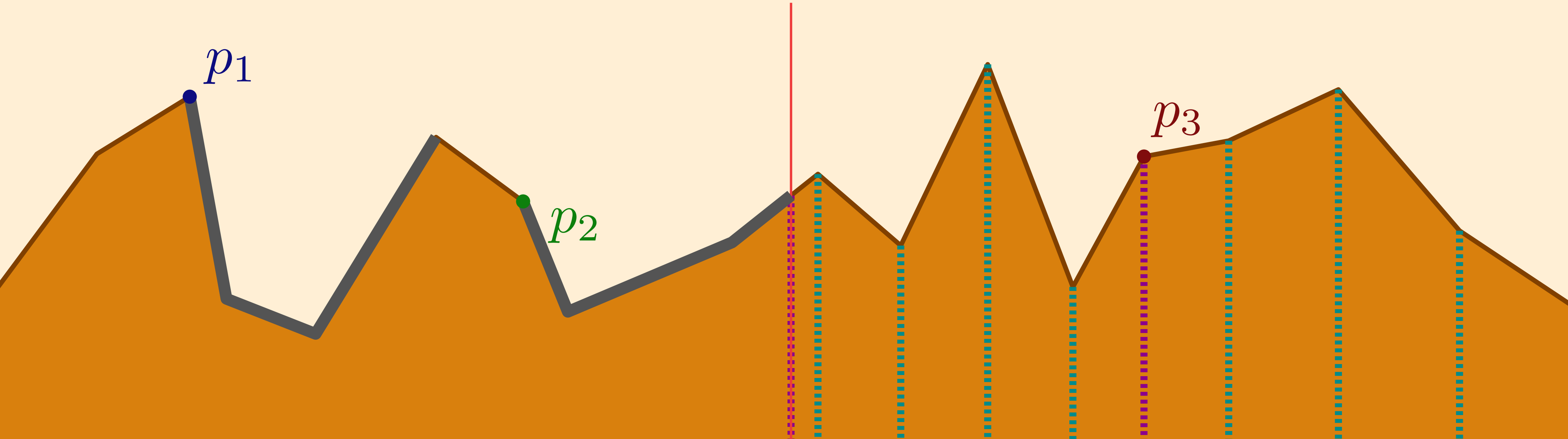
We compute *left* Vis:

$q \in \textit{left}\,\mathrm{Vis} \iff q$ visible by a viewpoint to the left of q.

Algorithm: Sweep the terrain, maintain leftmost visible viewpoint p_a.

- Viewpoint events
- Vertex events

p_a: p_1



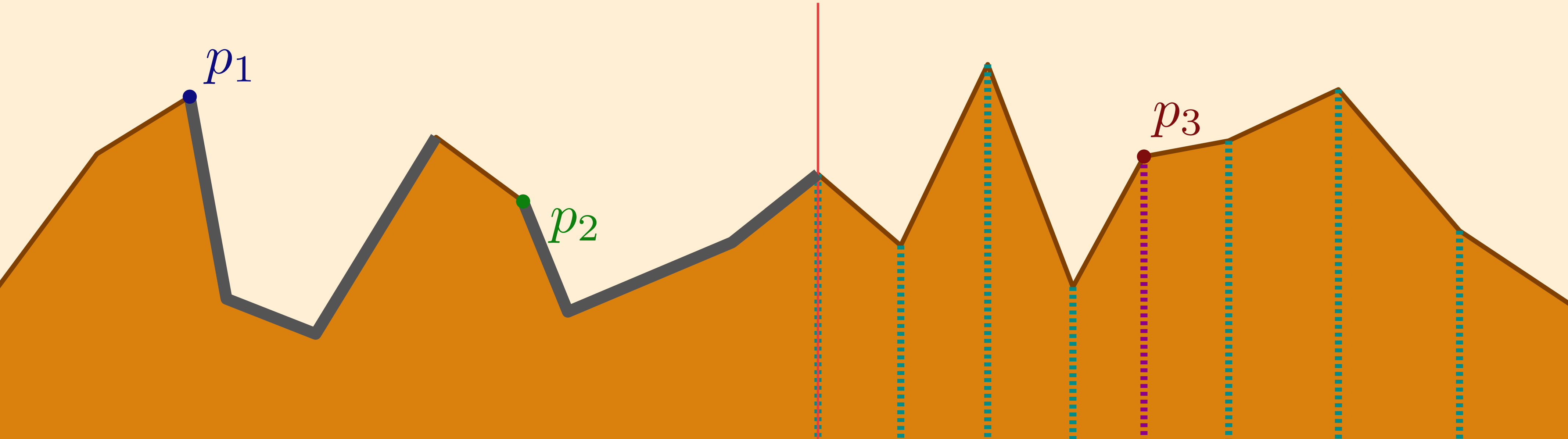
Computing Vis

We compute *left* Vis:
$q \in \mathit{left}\,\mathrm{Vis} \iff q$ visible by a viewpoint to the left of q.

Algorithm: Sweep the terrain, maintain leftmost visible viewpoint p_a.

- Viewpoint events
- Vertex events

p_a: p_1

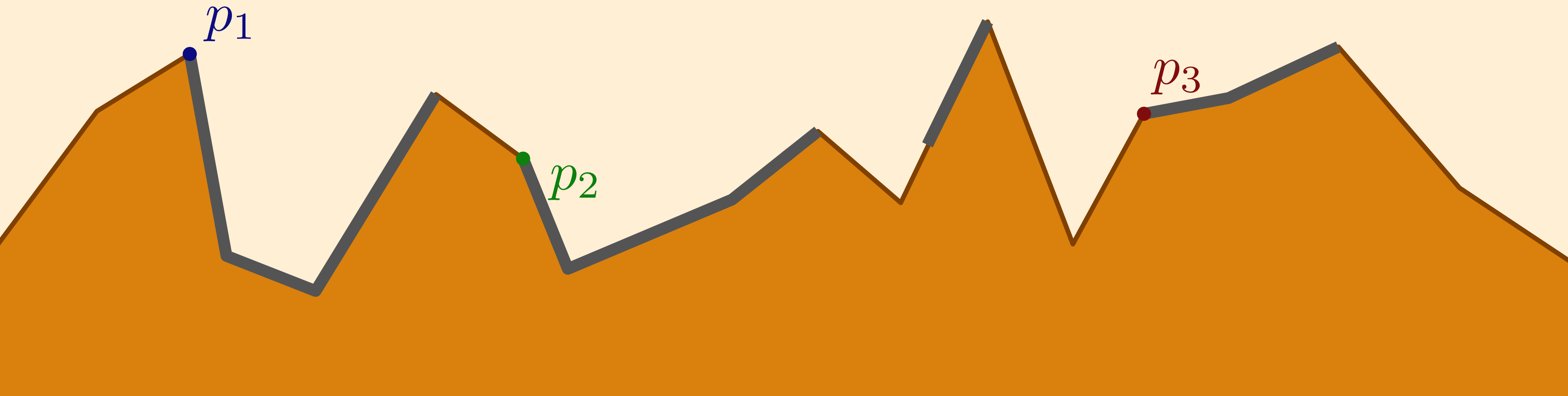


Computing Vis

We compute *left* Vis:
$q \in \mathit{left}\,\mathrm{Vis} \iff q$ visible by a viewpoint to the left of q.

Algorithm: Sweep the terrain, maintain leftmost visible viewpoint p_a.

- Viewpoint events
- Vertex events

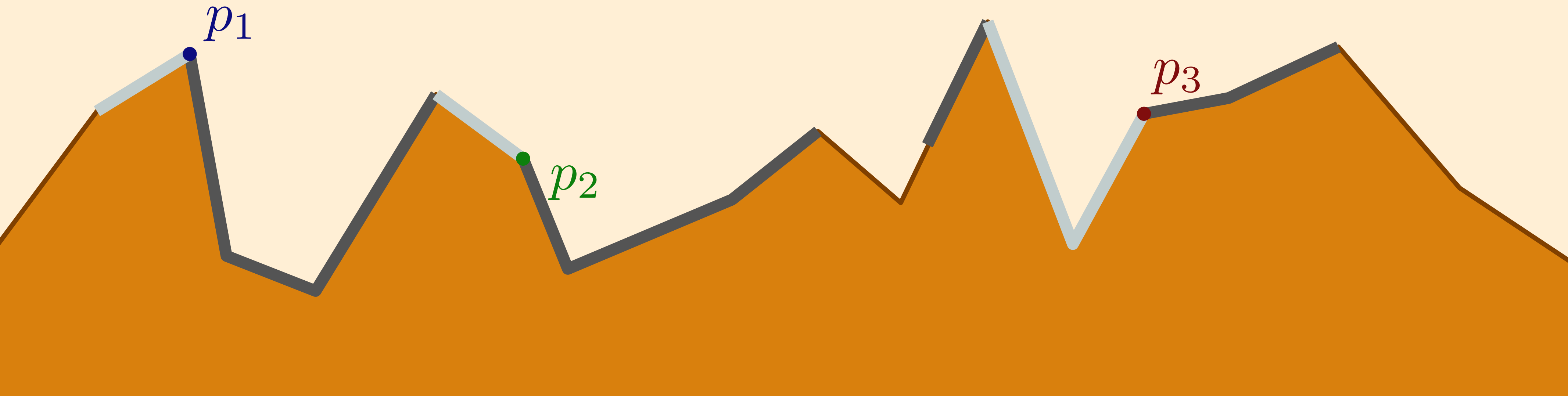


Computing Vis

We compute *left* Vis:
$q \in \textit{left}\,\mathrm{Vis} \iff q$ visible by a viewpoint to the left of q.

Algorithm: Sweep the terrain, maintain leftmost visible viewpoint p_a.

- Viewpoint events
- Vertex events

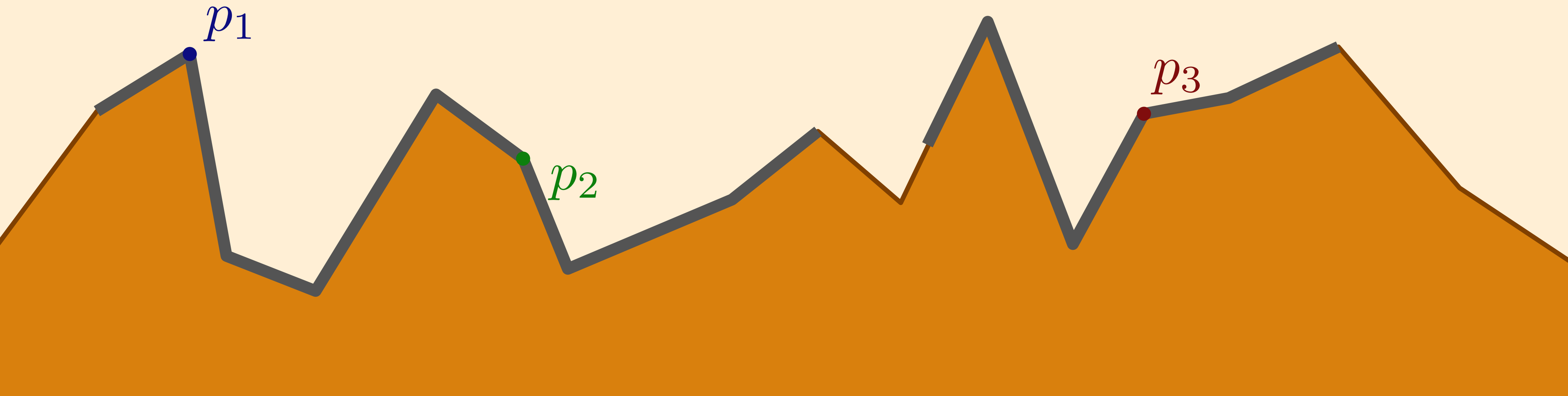


Computing Vis

We compute *left* Vis:
$q \in \textit{left}\,\mathrm{Vis} \iff q$ visible by a viewpoint to the left of q.

Algorithm: Sweep the terrain, maintain leftmost visible viewpoint p_a.

- Viewpoint events
- Vertex events



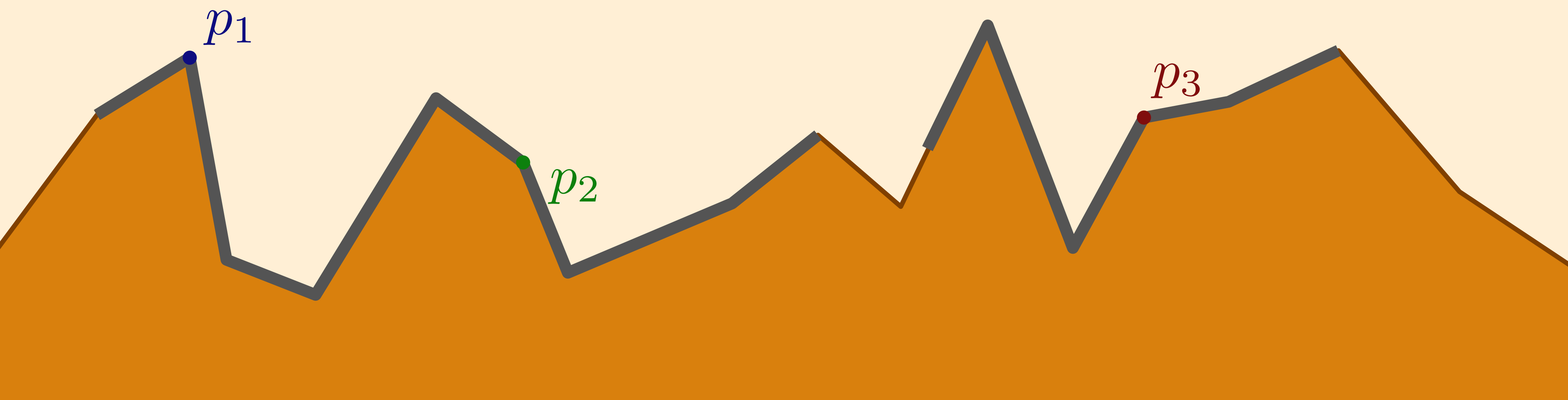
Computing Vis

We compute *left* Vis:
$q \in \mathit{left}\,\mathrm{Vis} \iff q$ visible by a viewpoint to the left of q.

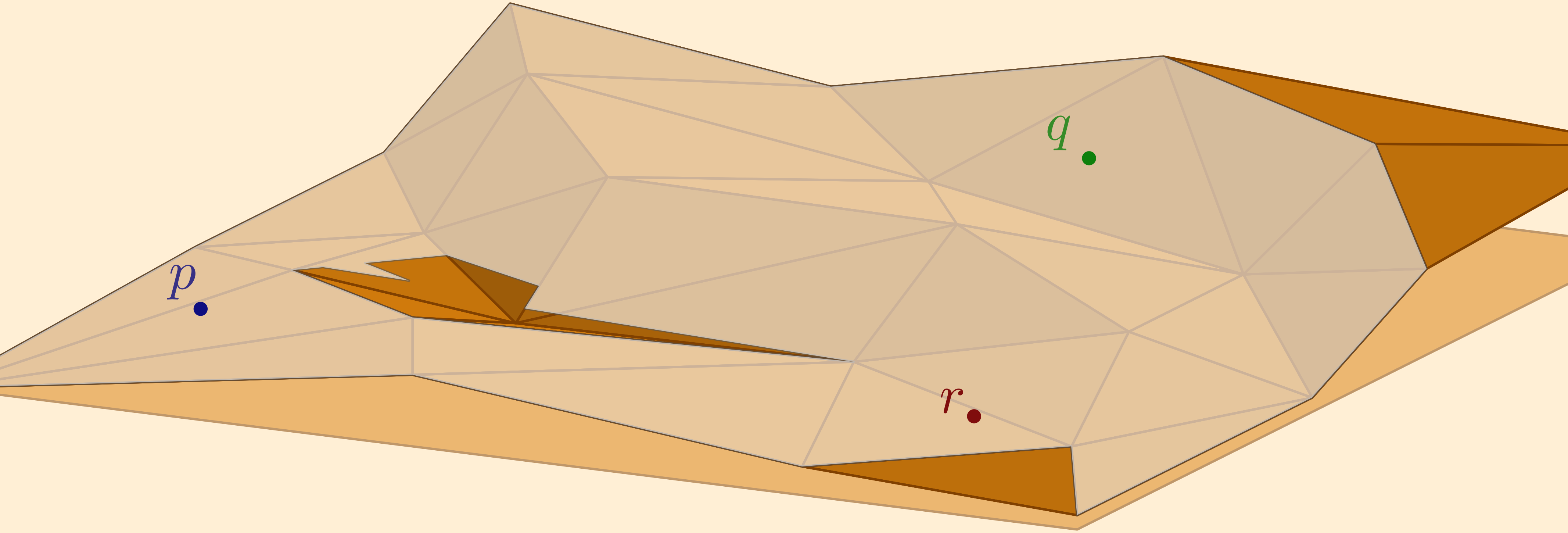
Algorithm: Sweep the terrain, maintain leftmost visible viewpoint p_a.

- Viewpoint events
- Vertex events

Running time: $O(\log n)$ time per event $\implies$ $O((k+n)\log n) = O(n \log n)$ time in total.

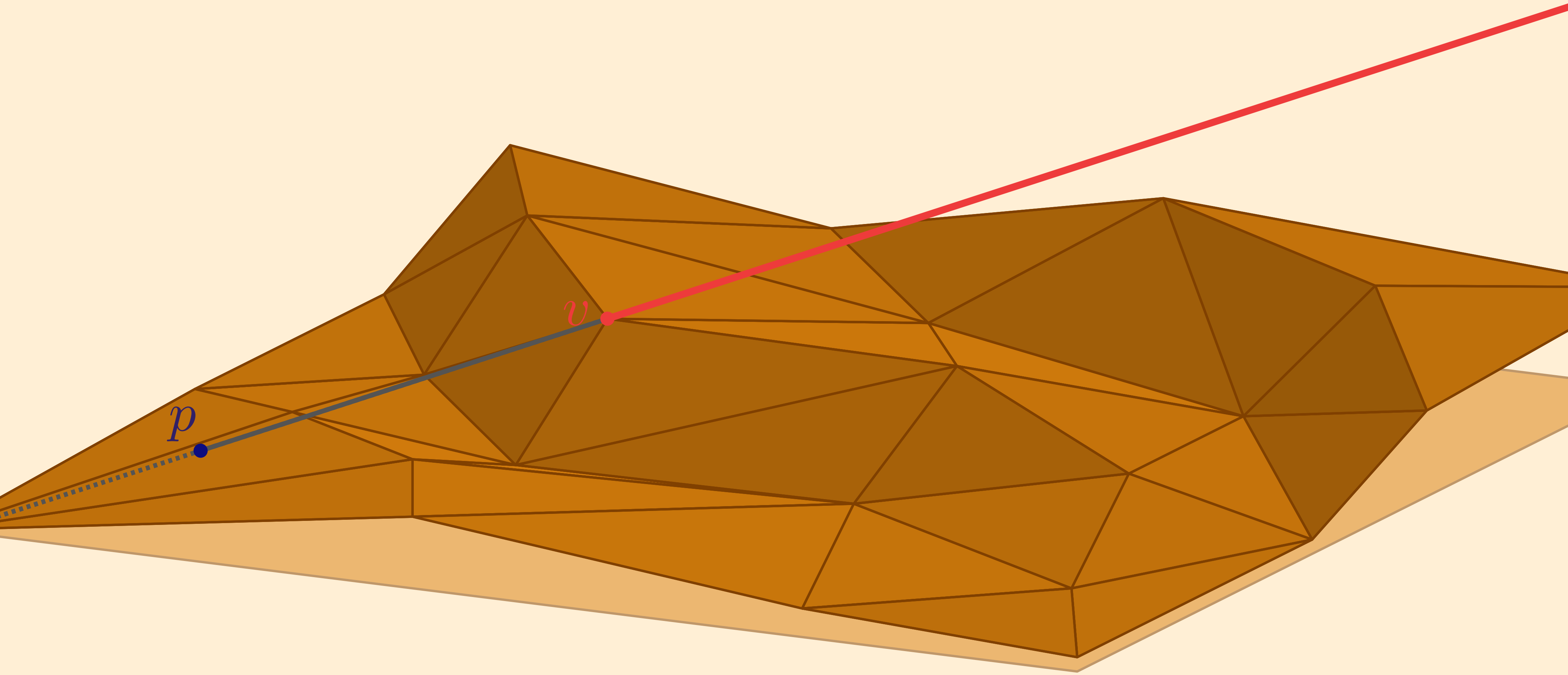


Complexity Vis



Complexity Vis

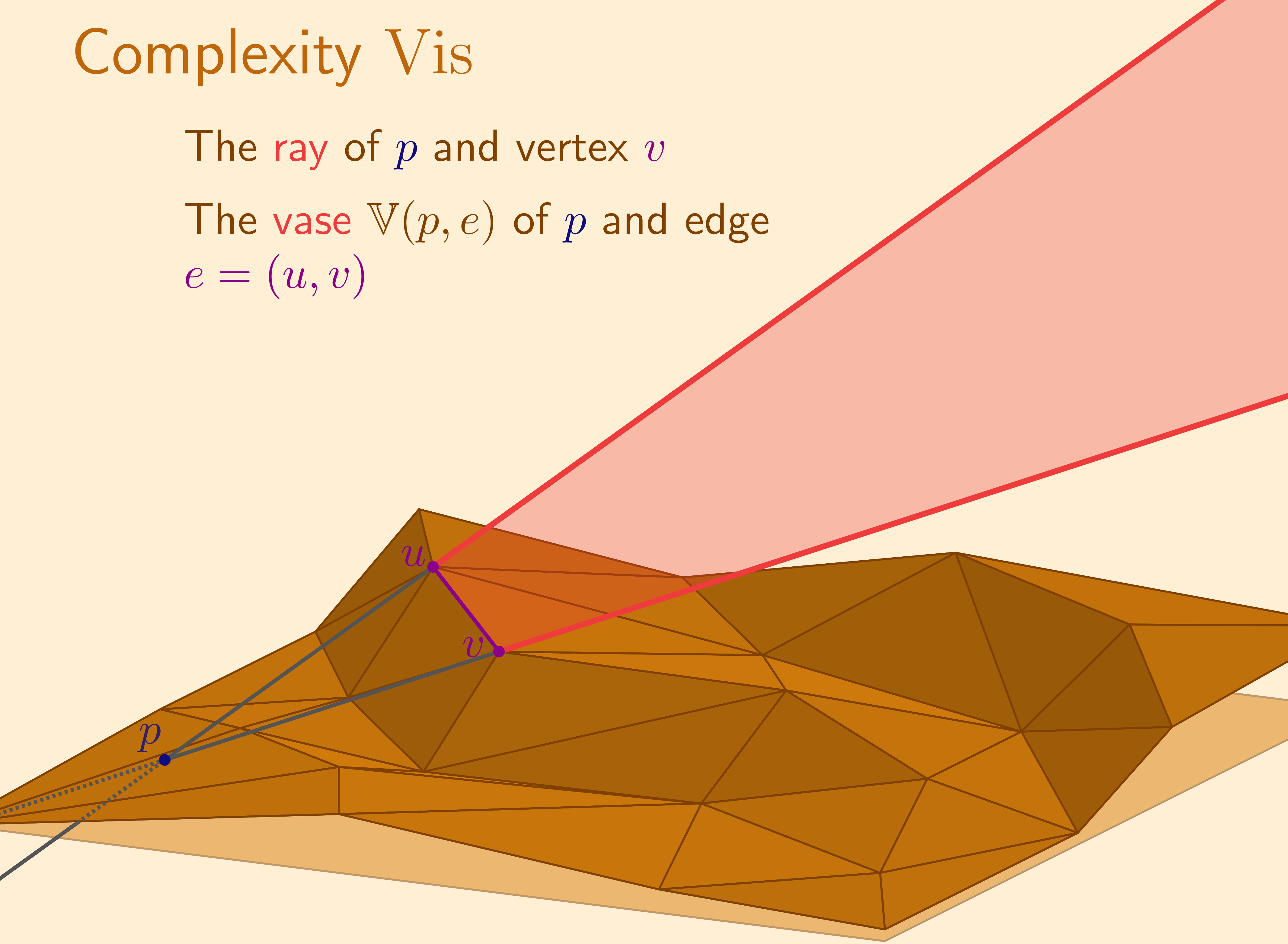
The ray of p and vertex v



Complexity Vis

The ray of p and vertex v

The vase $\mathbb{V}(p, e)$ of p and edge $e = (u, v)$

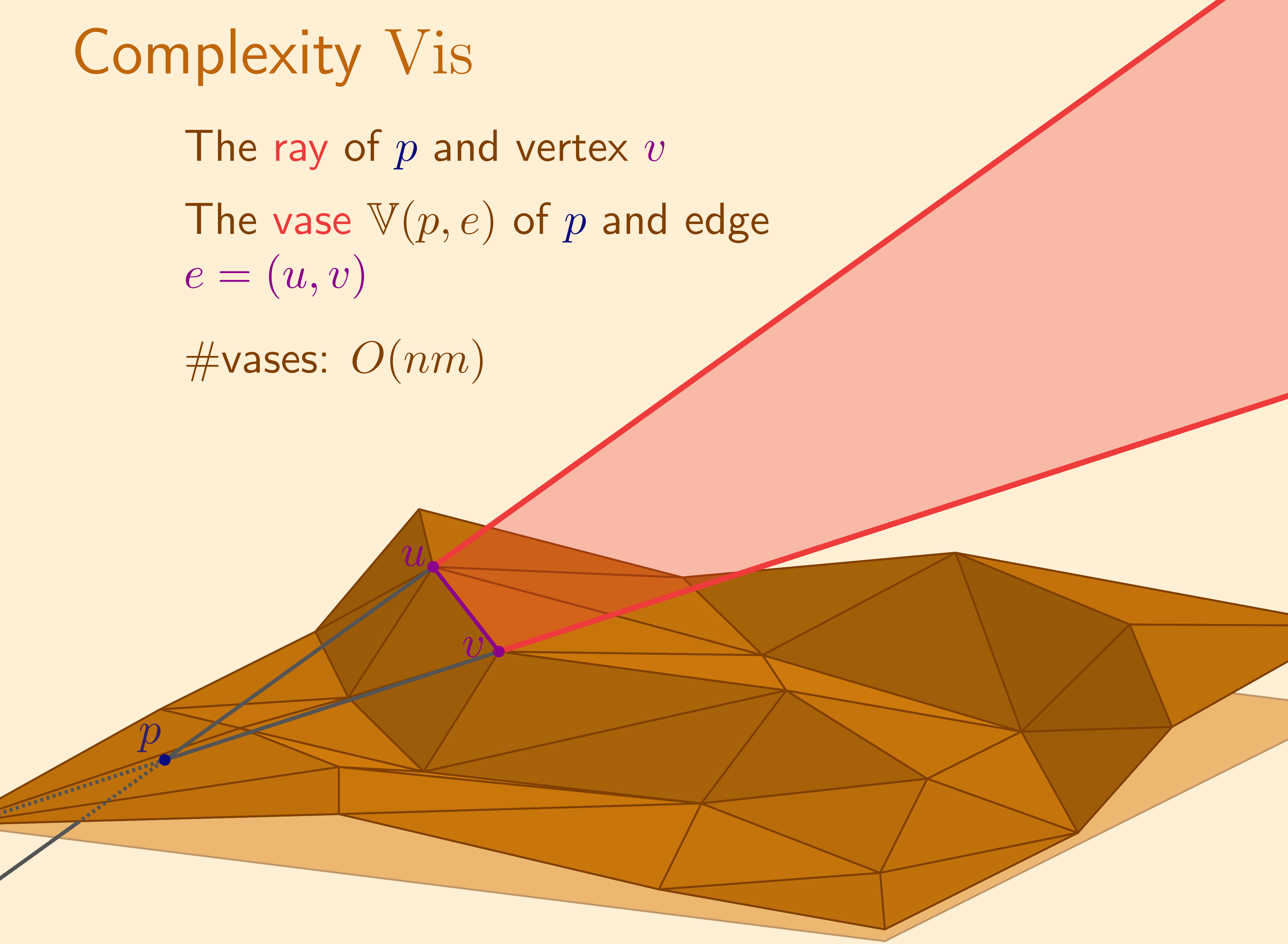


Complexity Vis

The ray of p and vertex v

The vase $\mathbb{V}(p, e)$ of p and edge $e = (u, v)$

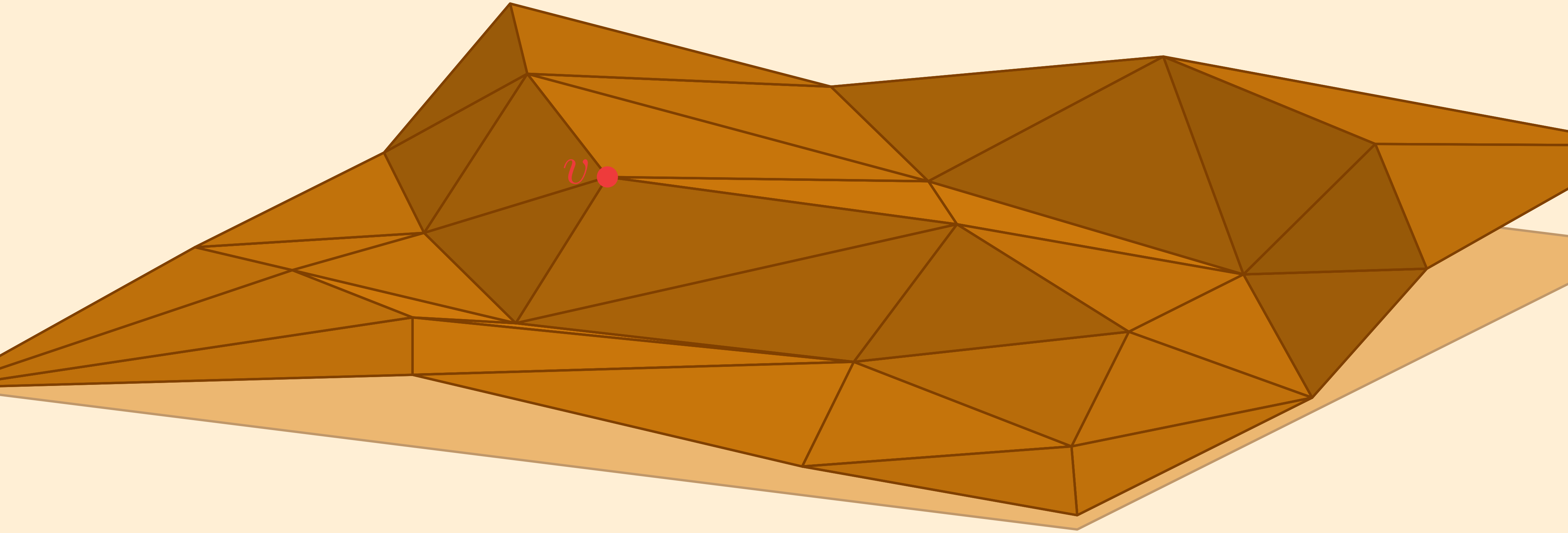
#vases: $O(nm)$



Complexity Vis

Classify the vertices of $\mathrm{Vis}(\mathcal{T}, \mathcal{P})$:

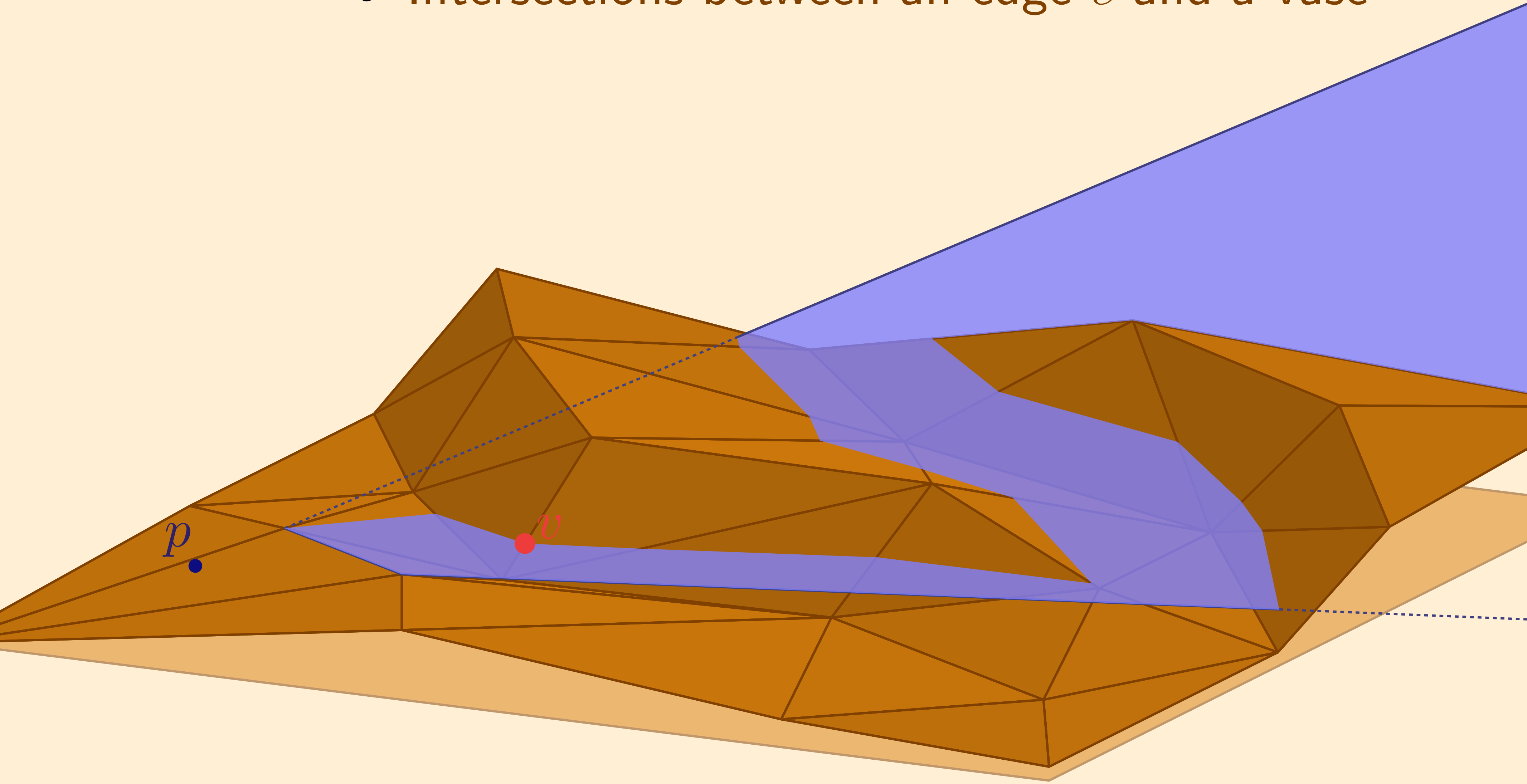
- vertices of $\mathcal{T}$



Complexity Vis

Classify the vertices of $\mathrm{Vis}(\mathcal{T}, \mathcal{P})$:

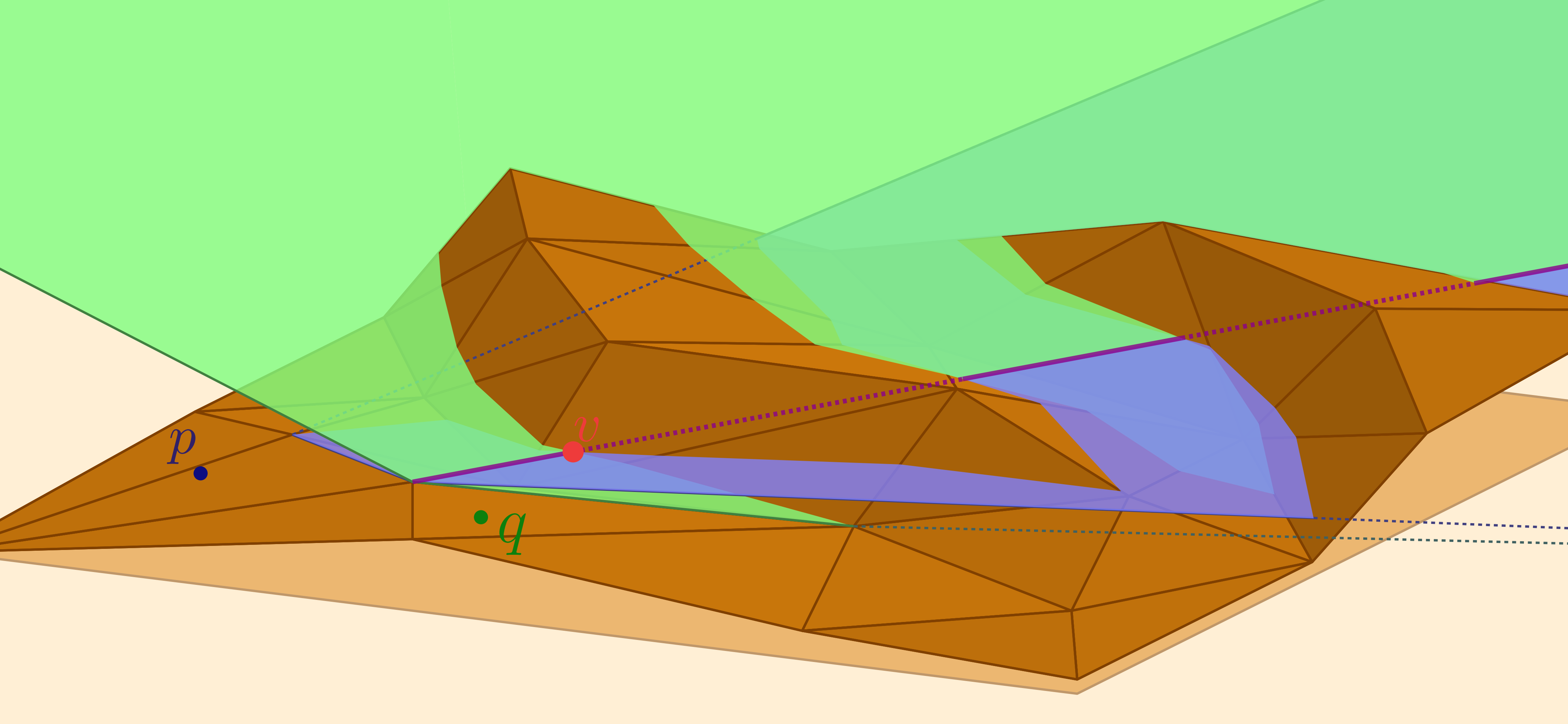
- vertices of $\mathcal{T}$
- Intersections between an edge e and a vase



Complexity Vis

Classify the vertices of $\mathrm{Vis}(\mathcal{T}, \mathcal{P})$:

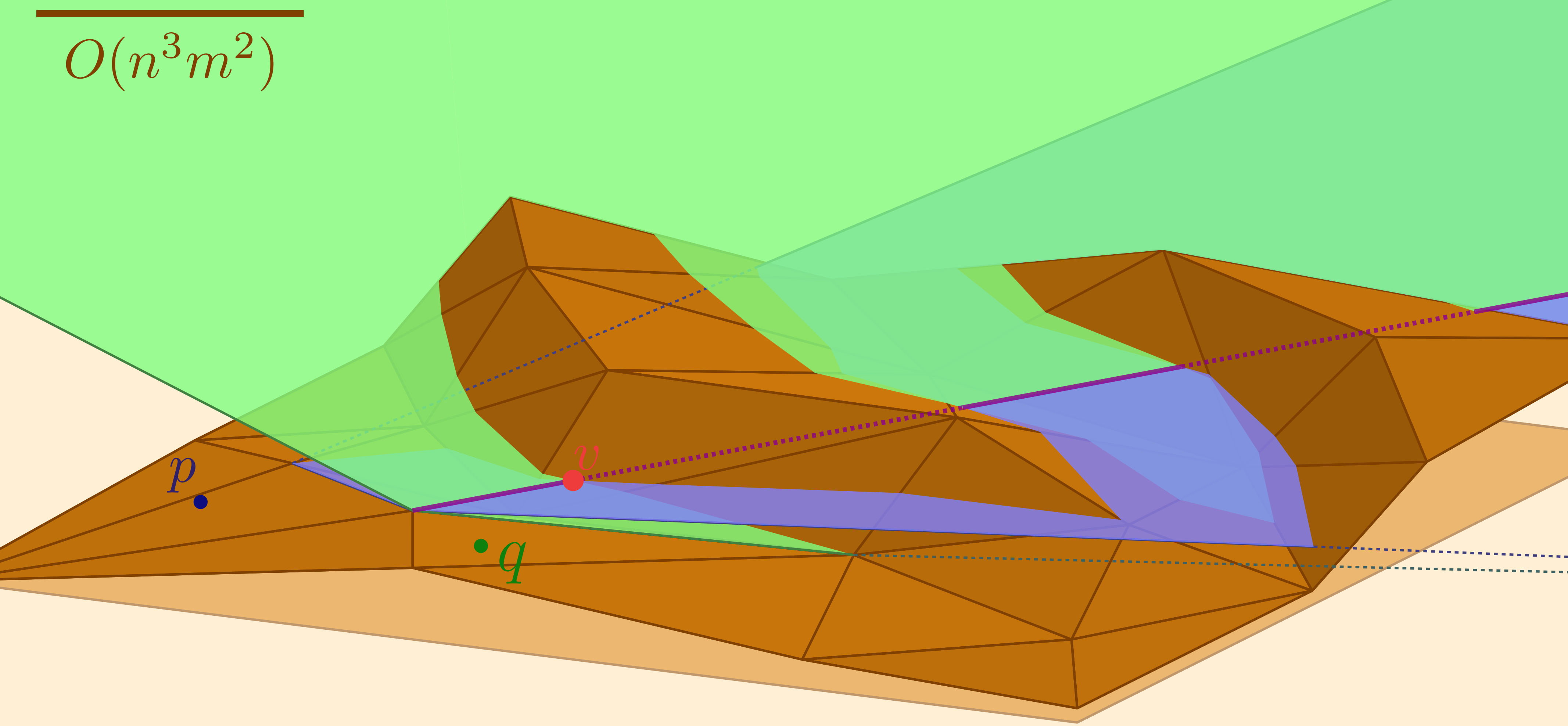
- vertices of $\mathcal{T}$
- Intersections between an edge e and a vase
- Intersections between a triangle t and 2 vases



Complexity Vis

Classify the vertices of $\mathrm{Vis}(\mathcal{T}, \mathcal{P})$:

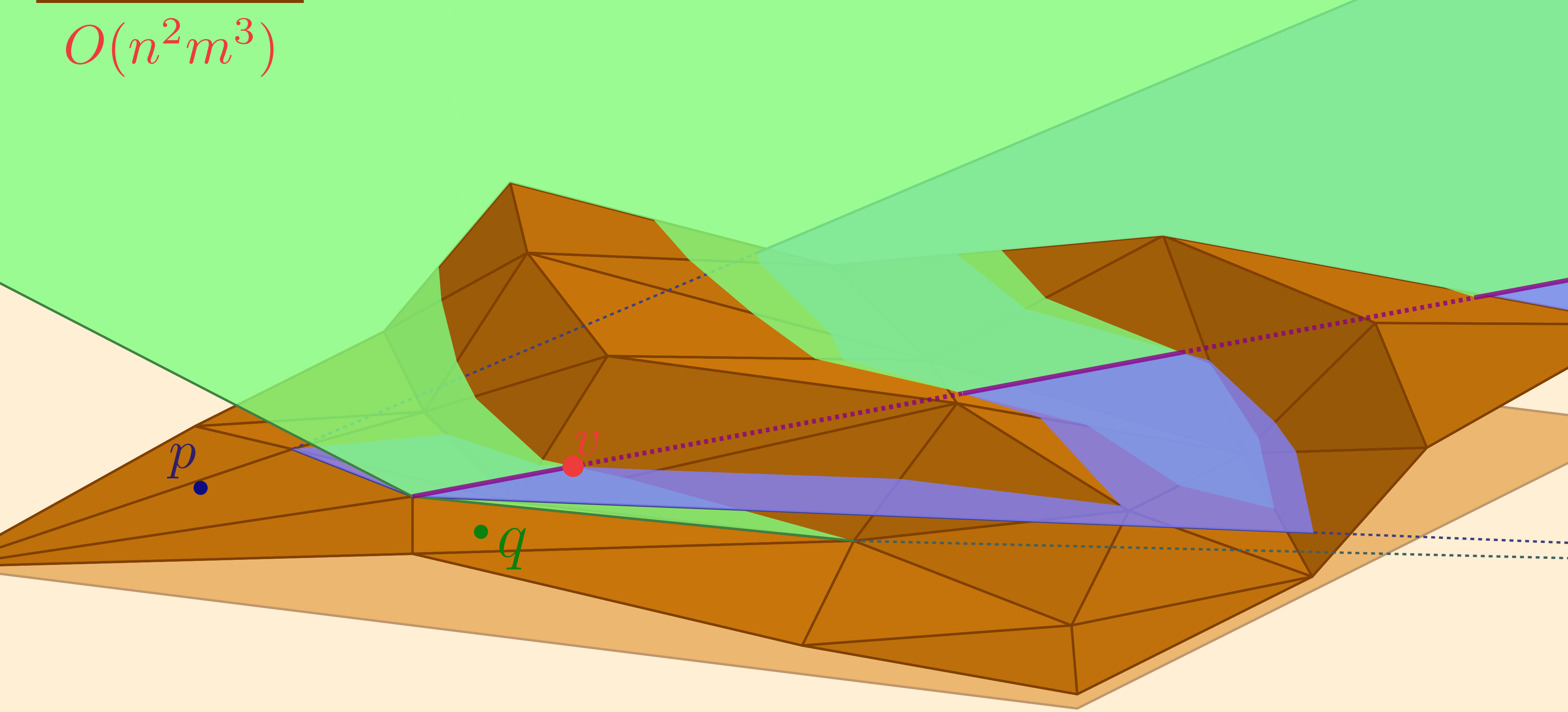
$O(n)$	• vertices of $\mathcal{T}$
$O(n^2m)$	• Intersections between an edge e and a vase
$O(n^3m^2)$	• Intersections between a triangle t and 2 vases
$O(n^3m^2)$	



Complexity Vis

Classify the vertices of $\mathrm{Vis}(\mathcal{T}, \mathcal{P})$:

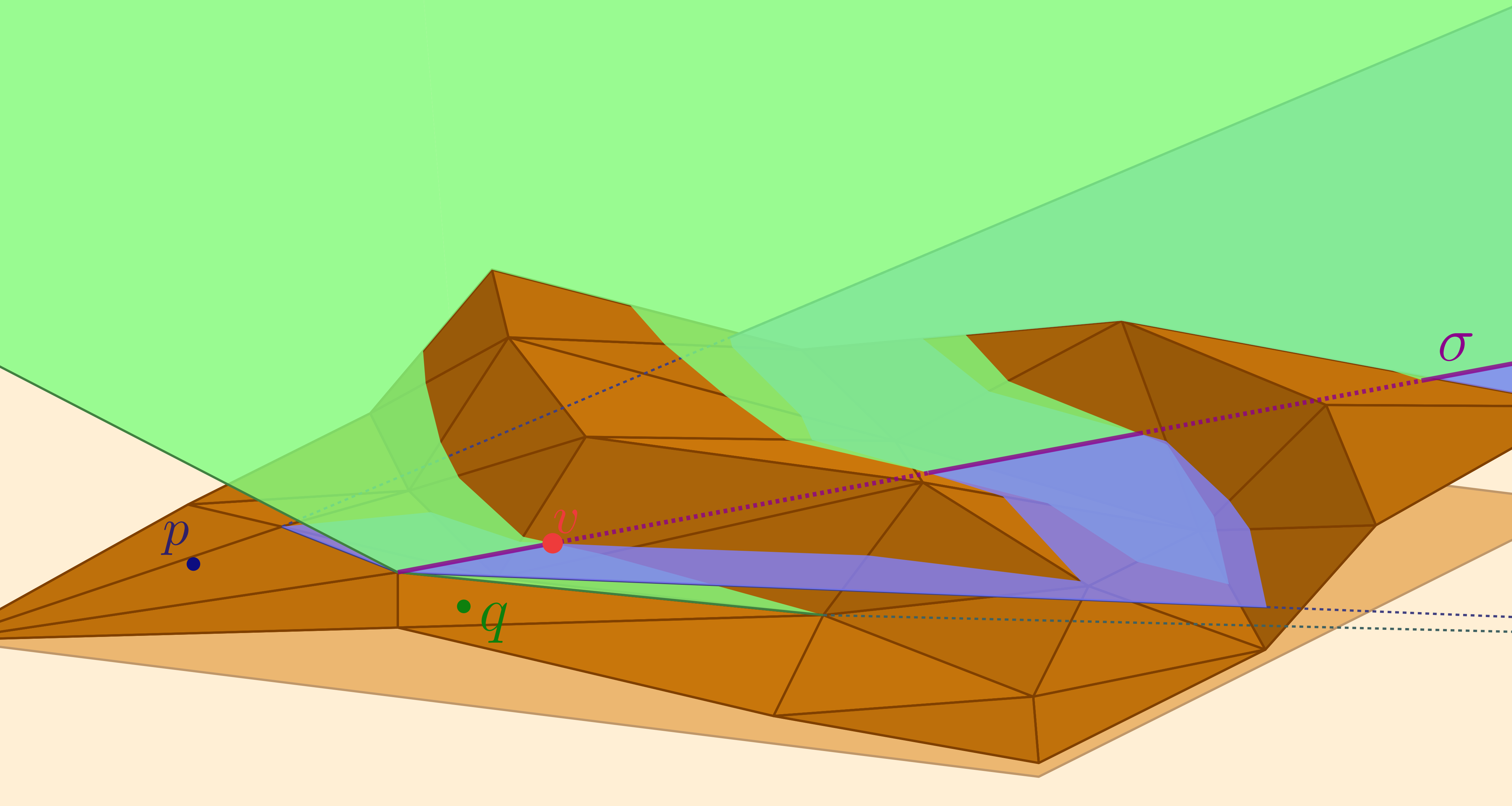
$O(n)$	vertices of $\mathcal{T}$
$O(n^2m)$	Intersections between an edge e and a vase
$O(n^2m^3)$	Intersections between a triangle t and 2 vases
$O(n^2m^3)$	



Complexity Vis

Bound #intersections between a triangle and 2 vases

Fix two vases, and let $\sigma = \mathbb{V}(p, e_p) \cap \mathbb{V}(q, e_q)$

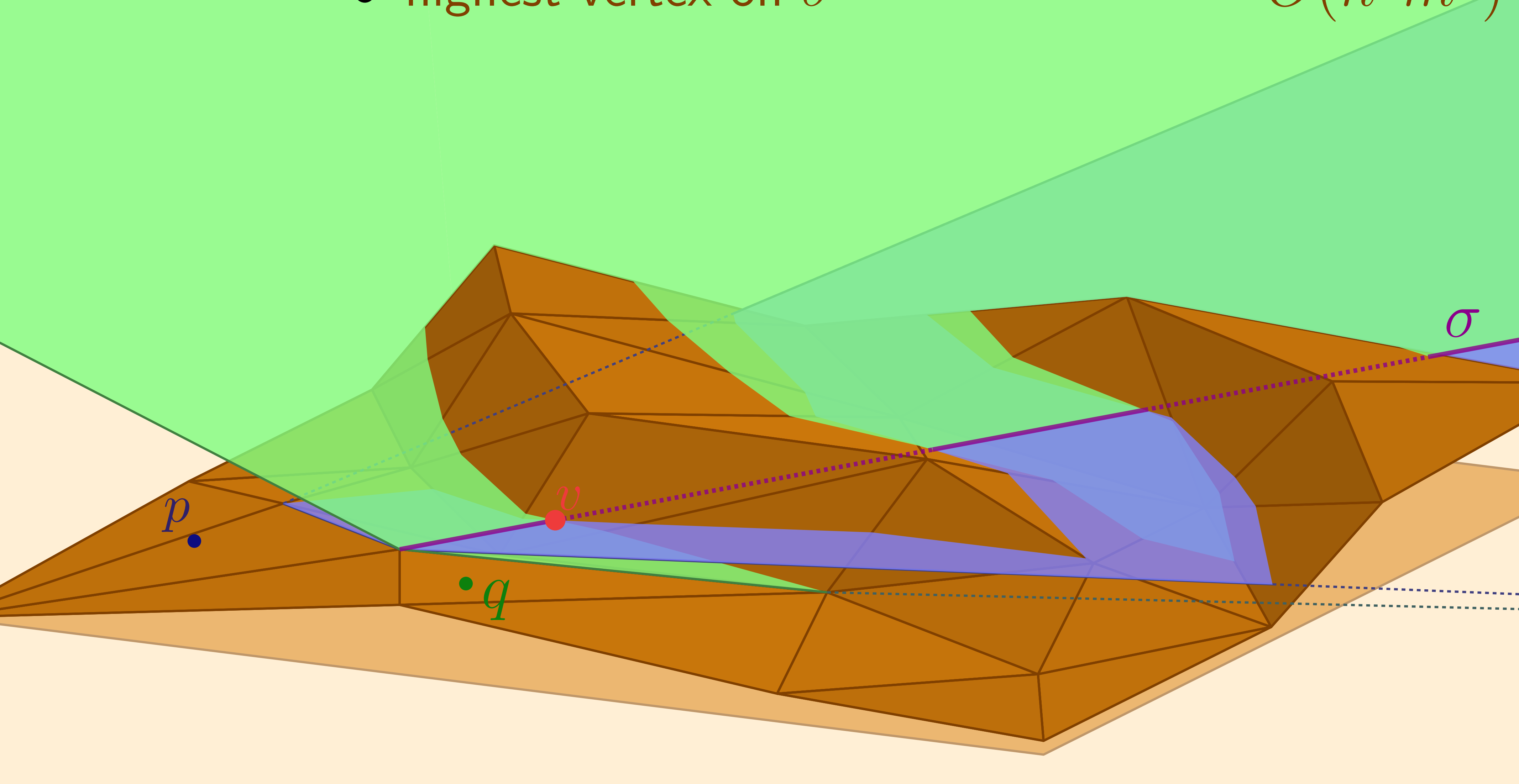


Complexity Vis

Bound #intersections between a triangle and 2 vases

Fix two vases, and let $\sigma = \mathbb{V}(p, e_p) \cap \mathbb{V}(q, e_q)$

- highest vertex on σ $O(n^2m^2)$

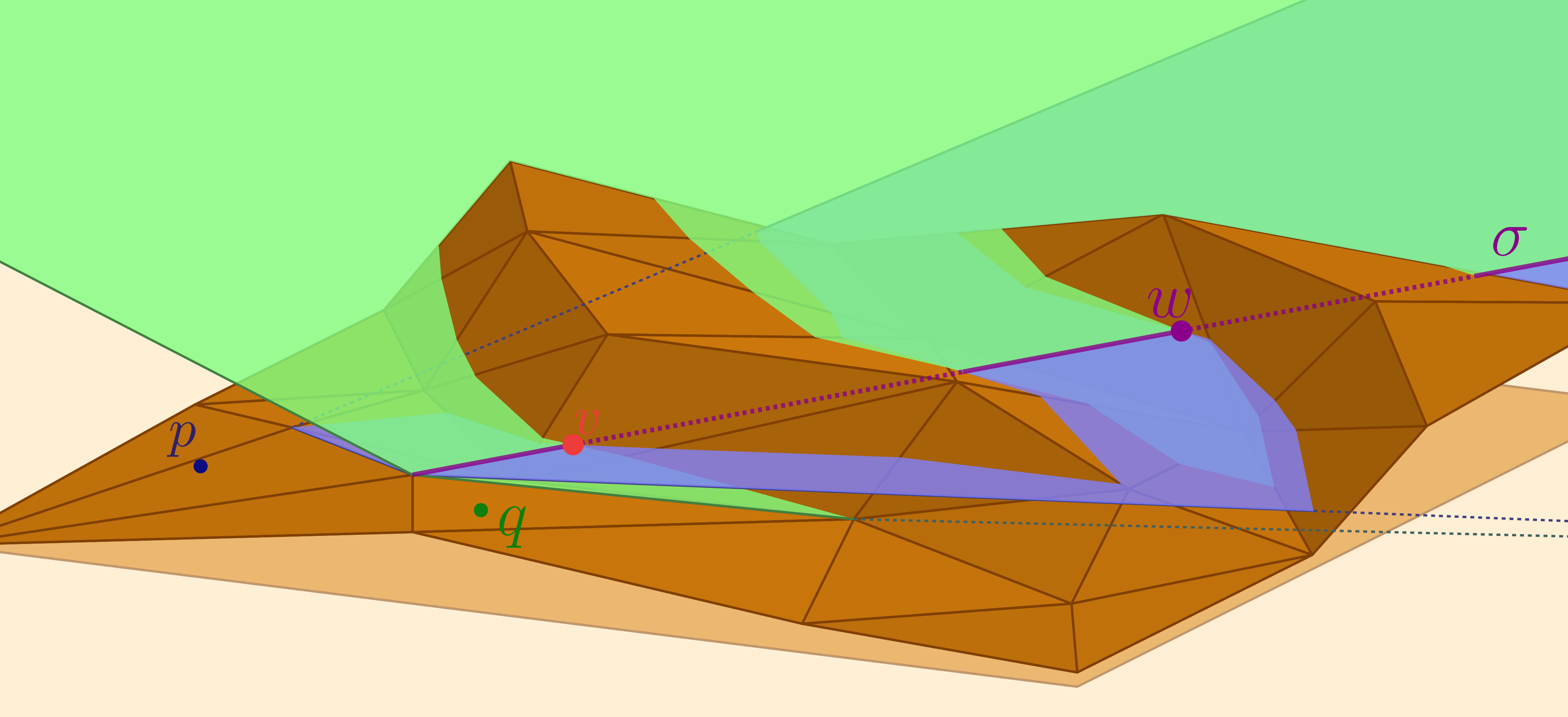


Complexity Vis

Bound #intersections between a triangle and 2 vases

Fix two vases, and let $\sigma = \mathbb{V}(p, e_p) \cap \mathbb{V}(q, e_q)$

- highest vertex on σ $O(n^2m^2)$
- otherwise

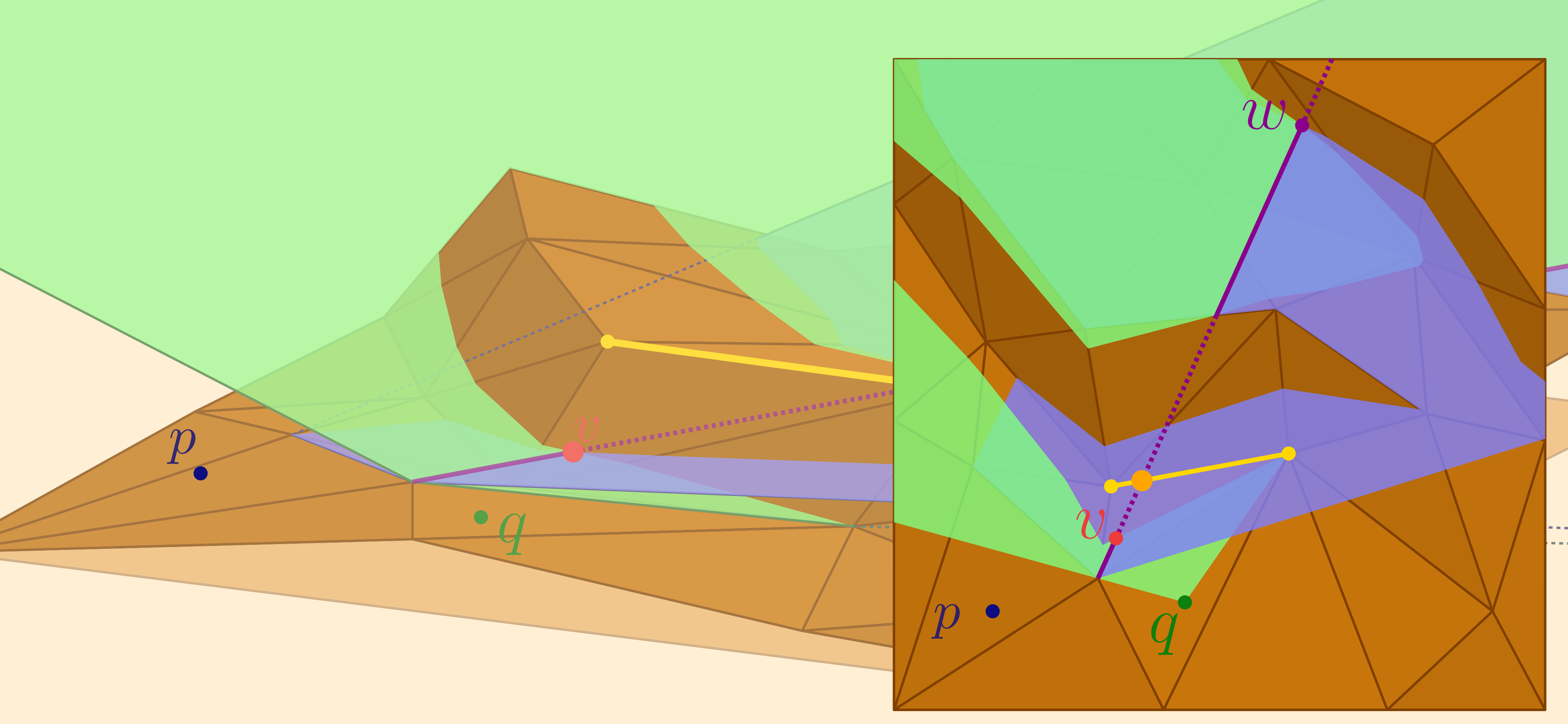


Complexity Vis

Bound #intersections between a triangle and 2 vases

Fix two vases, and let $\sigma = \mathbb{V}(p, e_p) \cap \mathbb{V}(q, e_q)$

- highest vertex on σ $O(n^2m^2)$
- otherwise charge intersection points



Complexity Vis

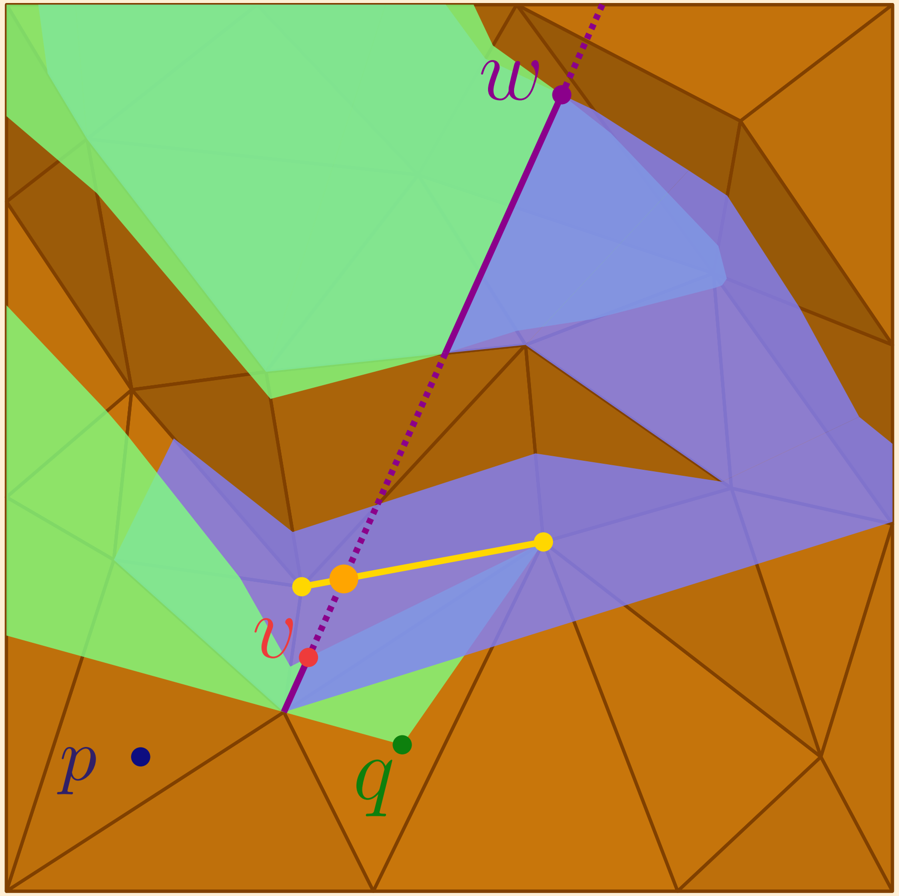
Bound #intersections between a triangle and 2 vases

Fix two vases, and let $\sigma = \mathbb{V}(p, e_p) \cap \mathbb{V}(q, e_q)$

- highest vertex on σ $O(n^2m^2)$
- otherwise charge intersection points

#intersection points?

Lemma 2. *Every edge intersects at most $O(nm^3)$ segments $\overline{vw}$.*



Complexity Vis

Bound #intersections between a triangle and 2 vases

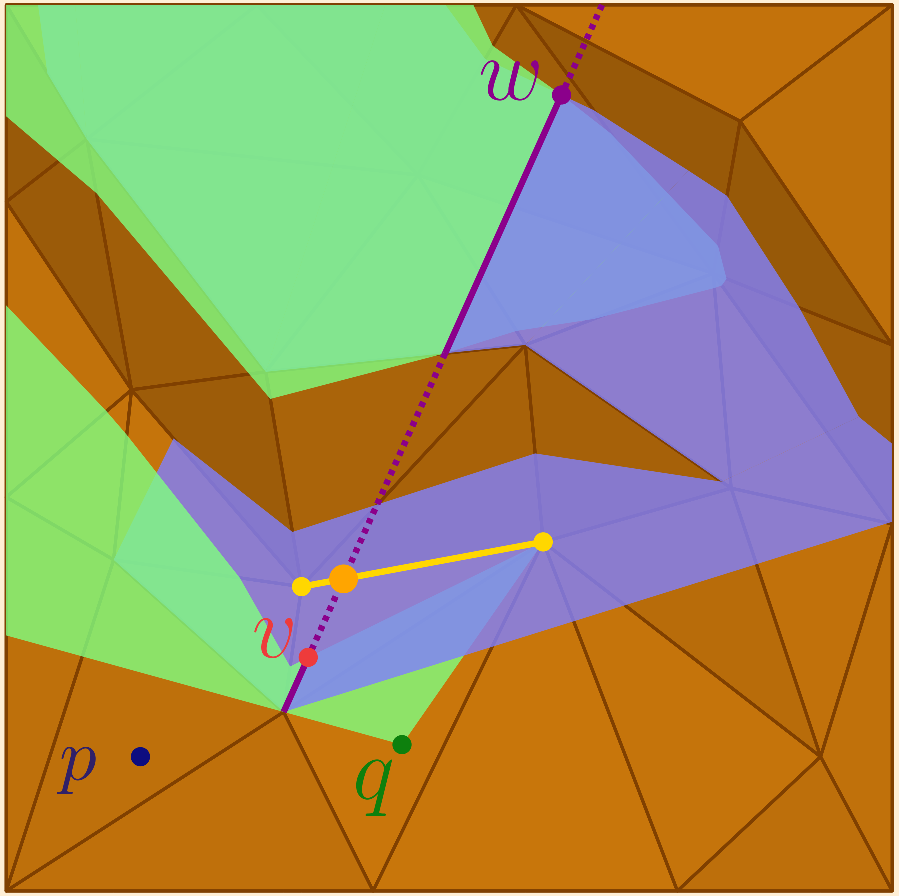
Fix two vases, and let $\sigma = \mathbb{V}(p, e_p) \cap \mathbb{V}(q, e_q)$

- highest vertex on σ $O(n^2m^2)$
- otherwise charge intersection points

#intersection points?

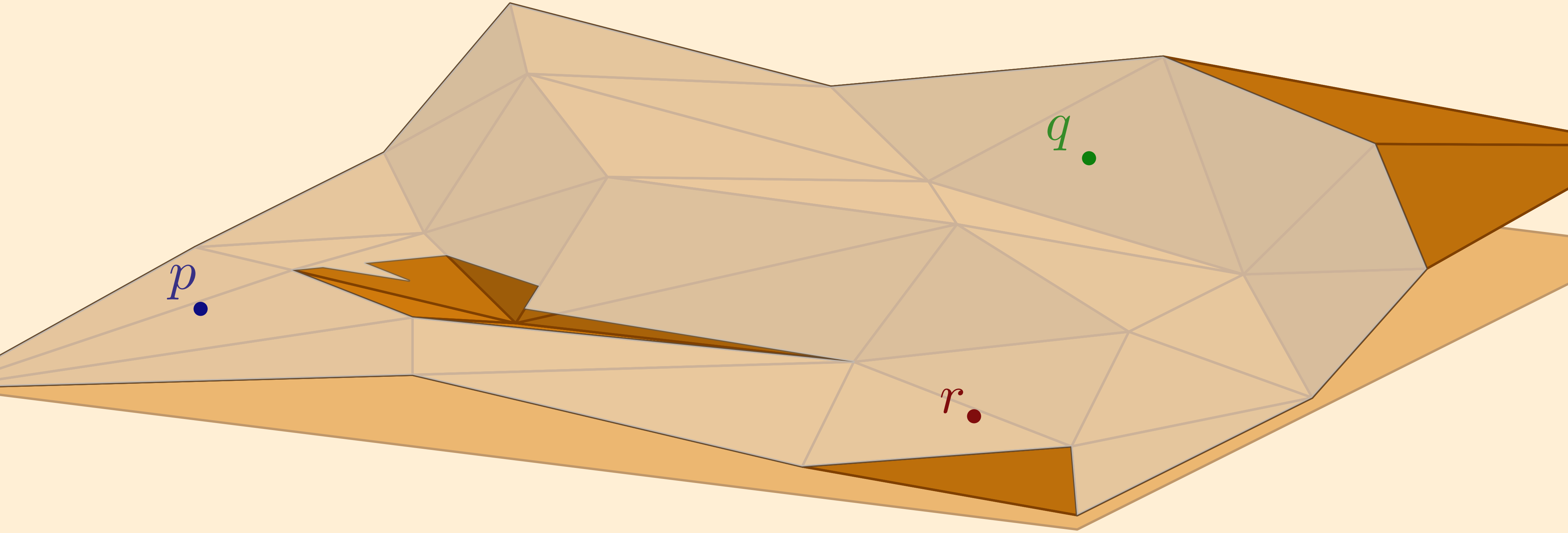
Lemma 2. *Every edge intersects at most $O(nm^3)$ segments $\overline{vw}$.*

Total: $O(n^2m^3)$



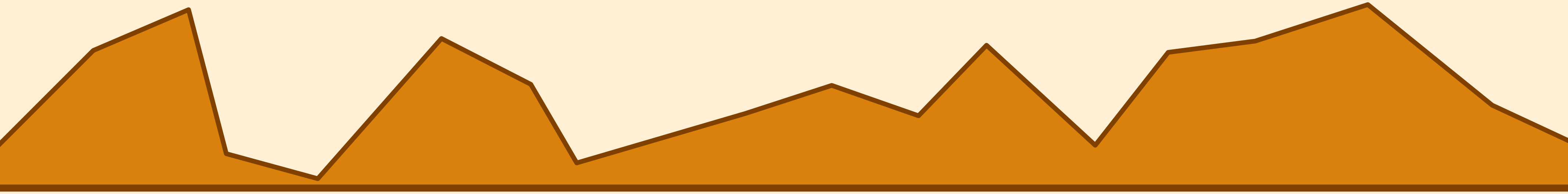
Complexity Vis

Theorem 3. *Complexity* Vis *is* $O(n^2m^3)$.



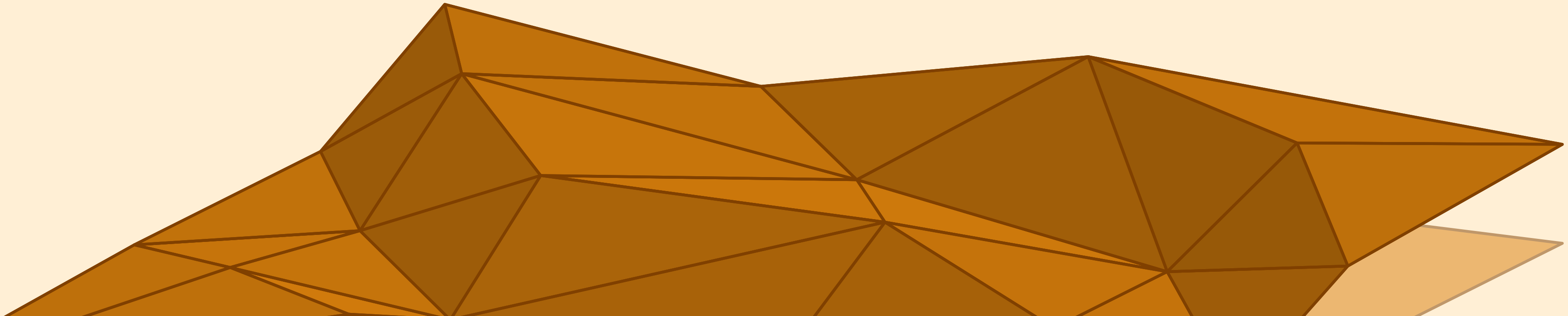
Future Work

VorVis algorithm depends on k_c.



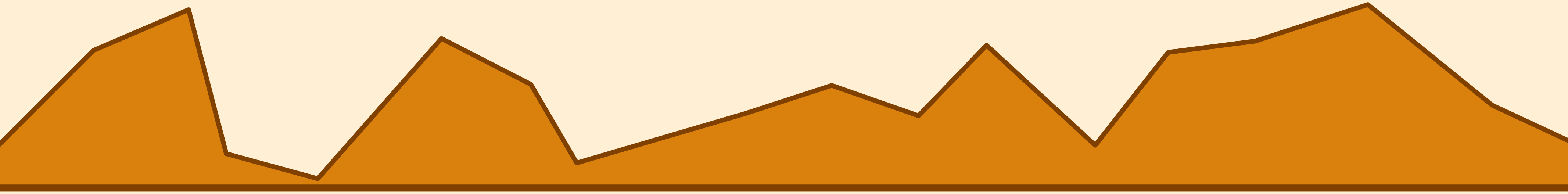
Improve complexity bounds.

Improve algorithms!



Future Work

VorVis algorithm depends on k_c.



Improve complexity bounds.

Improve algorithms!

Thank you!